$e^{\alpha x} \in \Omega_0 \qquad \alpha = ?$

$$x\alpha^3 + (5-6x)\alpha^2 + (12x-20)\cdot\alpha + 20 - 8x \overset{!}{=} 0$$

$$x(\alpha^3 - 6\alpha^2 + 12\alpha - 8) + 5\alpha^2 - 20\alpha + 20 = 0$$

$$x(\alpha-2)^3 + 5(\alpha-2)^2 = 0 \quad \Rightarrow \quad \alpha = 2 \quad \Rightarrow \quad e^{2x} \in \Omega_0 \ !$$

Snížení řádu: $y(x) = z(x)\cdot e^{2x} \qquad y' = z'e^{2x} + 2ze^{2x}$

$$y'' = z''e^{2x} + 4z'e^{2x} + 4ze^{2x}$$

$$y''' = z'''e^{2x} + 6z''e^{2x} + 12z'e^{2x} + 8ze^{2x}$$

Dosazení: $x(z''' + 6z'' + 12z') + (5-6x)(z'' + 4z') + (12x-20)z' = 0$

$$xz''' + z''(6x - 6x + 5) + z'(12\cancel{x} + 20 - 24x + 12x - 20) = 0$$

$$xz''' + 5z'' = 0$$

$$z''' + \frac{5}{x}z'' = 0 \qquad \Big| \; x^5 \quad \text{IF}$$

$$(z''\cdot x^5)' = C$$

$$z'' = \frac{C}{x^5} \Rightarrow z' = \frac{D}{x^4} + E \Rightarrow z = \frac{G}{x^3} + Ex + H$$

$$y(x) = H\cdot e^{2x} + E\cdot x\cdot e^{2x} + \frac{G}{x^3}e^{2x}$$

$$\mathrm{Dom}(y) = (0, +\infty)$$

• monom: x^m $(m \in \mathbb{N}_0)$

$$\langle x | x^m \rangle = \int_0^1 x \cdot x^m \, dx = \left[\frac{x^{m+2}}{m+2} \right]_0^1 = \frac{1}{m+2}$$

$$\langle x | x \rangle = \| x \|^2 = \int_0^1 x^2 \, dx = \frac{1}{3} \Rightarrow \| x \| = \frac{1}{\sqrt{3}}$$

$$\langle x^m | x^m \rangle = \| x^m \|^2 = \int_0^1 x^{2m} \, dx = \frac{1}{2m+1} \Rightarrow \| x^m \| = \frac{1}{\sqrt{2m+1}}$$

vzorec pro úhel

$$\cos 30^\circ = \frac{\frac{1}{m+2}}{\frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{2m+1}}}$$

$$\frac{\sqrt{3}}{2} = \frac{\sqrt{3} \cdot \sqrt{2m+1}}{m+2}$$

$$m+2 = 2\sqrt{2m+1}$$

$$m^2 + 4m + 4 = 8m + 4$$

$$m^2 - 4m = 0$$

$$m(m-4) = 0$$

Dvě řešení:

$$g_1(x) = 1 \quad \& \quad g_2(x) = x^4$$

In[18]:= **Expand[(x + y + z - 2)^2 - (y - 2 z + 1)^2 - (z + 2)^2 + 1]**

Out[18]= $-4x + x^2 - 6y + 2xy - 4z + 2xz + 6yz - 4z^2$

$$(x+y+z)^2 + 4yz - y^2 - 5z^2 - 4x - 6y - 4z = 0$$

$$\underbrace{(x+y+z)}_{\alpha}^2 - \underbrace{(y-2z)}_{\beta}^2 - \underbrace{z}_{\gamma}^2 - 4x - 6y - 4z = 0$$ úpravení na čtverce

$$\left.\begin{aligned} \alpha &= x+y+z \\ \beta &= y-2z \\ \gamma &= z \end{aligned}\right\} \Rightarrow \begin{aligned} x &= \alpha - \beta - 2\gamma - \gamma = \alpha - \beta - 3\gamma \\ y &= \beta + 2\gamma \\ z &= \gamma \end{aligned}$$

$$\alpha^2 - \beta^2 - \gamma^2 - 4\alpha + \underline{4\beta} + 12\gamma \underline{- 6\beta} - 12\gamma - 4\gamma = 0$$

$$\alpha^2 - \beta^2 - \gamma^2 - 4\alpha - 2\beta - 4\gamma = 0$$

$$\underbrace{(\alpha - 2)}_{\lambda}^2 - \underbrace{(\beta + 1)}_{\mu}^2 - \underbrace{(\gamma + 2)}_{\omega}^2 = 4 - 1 - 4$$ úpravení na druhé čtverce

$$\lambda^2 - \mu^2 - \omega^2 = -1$$

$$\mu^2 + \omega^2 - \lambda^2 = 1$$ normální tvar

$sg(Q) = (2, 1, 0)$ $SG(Q) = (2, 2, 0)$ jednodílný el. hyperboloid

$$\left.\begin{aligned} \alpha &= \lambda + 2 \\ \beta &= \mu - 1 \\ \gamma &= \omega - 2 \end{aligned}\right\} \Rightarrow \begin{aligned} x &= \lambda + 2 - \mu + 1 - 3\omega + 6 = \lambda - \mu - 3\omega + 9 = x \\ y &= \mu - 1 + 2\omega - 4 \Rightarrow y = \mu + 2\omega - 5 \\ z &= \omega - 2 \Rightarrow z = \omega - 2 \end{aligned}$$

normalizující transf.

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & -1 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \\ \omega \end{pmatrix} + \begin{pmatrix} 9 \\ -5 \\ -2 \end{pmatrix}$$

střed

$$g_n(x) = \sqrt{\frac{x^2+n^2-n^2}{x^2+n^2}} = \frac{x}{\sqrt{x^2+n^2}}$$

$$g_n'(x) = \frac{1}{x^2+n^2}\left(\sqrt{x^2+n^2} - x\,\frac{\frac{1}{2}\cdot 2x}{\sqrt{x^2+n^2}}\right) = \overbrace{\frac{x^2+n^2-x^2}{(x^2+n^2)^{3/2}}}^{g_n'(x)} \overset{!}{=} 0$$

jenže tohle se nule nerovná nikdy!

$$g_n(0) = 0 \qquad g_n'(x) > 0 \text{ na } R$$

$$\Rightarrow \sup g_n(x) = \lim_{x\to+\infty} g_n(x) = \lim_{x\to\infty} \frac{x}{\sqrt{x^2+n^2}} = 1$$

$\Rightarrow \forall x \in R^+$:

$$\left|\frac{(-1)^{n+1}}{\sqrt{n}}\sqrt{1-\frac{n^2}{x^2+n^2}}\right| \leq \frac{1}{\sqrt{n}}$$

lepší horní odhad neexistuje!

Jenže řada $\sum_n \frac{1}{\sqrt{n}}$ diverguje $\Rightarrow$ Weierstrassovo kritérium o SK nerozhodne!

$$g(x) = (1-3x)^{-2/3} = \sum_{n=0}^{\infty} \binom{-2/3}{n} (-3x)^n =$$

$$= 1 + \sum_{n=1}^{\infty} \frac{(-\frac{2}{3})(-\frac{5}{3})\dots(-\frac{2}{3}-n+1)}{n!} (-3)^n x^n =$$

$$= 1 + \sum_{n=1}^{\infty} \frac{2\cdot 5\cdot 8 \dots (3n-1)}{n!} x^n =$$

$$= 1 + \sum_{n=1}^{\infty} \frac{(3n-1)!!!}{n!} x^n$$

$$R^{-1} = \lim_{n\to\infty} \frac{(3n+2)!!!}{(n+1)!} \frac{n!}{(3n-1)!!!} = \lim_{n\to\infty} \frac{3n+2}{n+1} = 3$$

$$R = 1/3$$

Interval konvergence: $I = \left(-\frac{1}{3}; \frac{1}{3}\right)$ ale to není obor konvergence

neto správné krajní body

krajní řady: $\sum_{n=1}^{\infty} (-1)^n \frac{(3n-1)!!!}{3^n \cdot n!}$ tvar krajních řad

Raabe:

$$\lim_{n\to\infty} n \left(1 - \frac{(3n+2)!!!}{3^{n+1}(n+1)!} \frac{3^n \cdot n!}{(3n-1)!!!}\right) =$$

$$\lim_{n\to\infty} n\left(1 - \frac{3n+2}{3(n+1)}\right) = \lim_{n\to\infty} n \frac{3n+3-3n-2}{3n+3} = \frac{1}{3}$$

$\frac{1}{3} \in (0,1)$, apodo

$$O = \left\langle -\frac{1}{3}; \frac{1}{3}\right)$$