(1) $\rho = 0 \Rightarrow v = 130 \ \mathrm{km/h}$

$\rho = \rho_J = 80 \ \#/\mathrm{km} \Rightarrow v = \dfrac{130}{e^8} \ \mathrm{km/h}$

6b

$$J = \rho \cdot v = A\rho\, e^{-B\rho^2} \Leftarrow v = A e^{-B\rho^2}$$

• umeritev konstant:

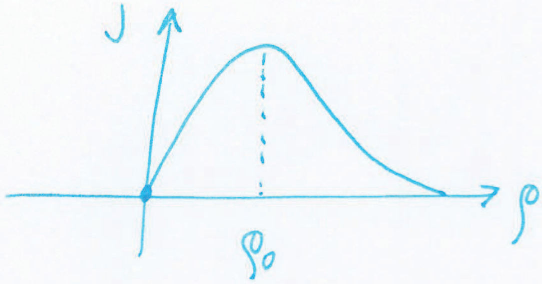
$$A = 130 \quad (\rho = 0)$$

$$\frac{130}{e^8} = 130 \cdot e^{-B\rho_J^2} \Rightarrow e^{-8} = e^{-B\rho_J^2} \Rightarrow \underline{\underline{B = \frac{8}{\rho_J^2}}}$$

$$\underline{\underline{v = 130 \cdot e^{-8\left(\frac{\rho}{\rho_J}\right)^2}}}$$

• tok

$$J = 130\rho\, e^{-8\left(\frac{\rho}{\rho_J}\right)^2}$$



$$\frac{\partial J}{\partial \rho} = 130\left(1 - 16\frac{\rho_0^2}{\rho_J^2}\right) e^{-8\left(\frac{\rho}{\rho_J}\right)^2} = 0$$

$$\underline{\underline{\rho_0 = \frac{\rho_J}{4} = 20 \ \#/\mathrm{km}}}$$

$$J_{max} = J(20) = 130 \cdot 20 \cdot e^{-8\left(\frac{20}{80}\right)^2} = 2600\, e^{-8 \cdot \frac{1}{16}} = 2600 \cdot e^{-1/2}$$

$$J_{max} = \frac{2600}{\sqrt{e}} = \frac{2600}{2} \approx \underline{\underline{1300}}$$

(2) 10b.

$p(x)$ .. hustota pravděpodobnosti odstupu

$g(v)$ .. — " — rychlostí

nezávislost

$\to f(x,v)$ .. sdružená hustota, $f(x,v) = g(v) \cdot p(x)$ ✓

- w .. fixní rychlost $\Rightarrow$ $\left| \begin{matrix} x = t \cdot w \\ v = w \end{matrix} \quad \det\left(\frac{D(x,v)}{D(t,w)}\right) = \begin{vmatrix} w & t \\ 0 & 1 \end{vmatrix} = w \right|$

$$\Rightarrow g(t,w) = f(wt, w) \cdot \left|\det\left(\frac{D(x,v)}{D(t,w)}\right)\right| = |w| f(wt,w) \approx$$

rychlost se nemění prudce

$$\approx v \cdot f(vt, v) = v \cdot g(v) \cdot p(v \cdot t)$$ ✓

- $$\Rightarrow J(t) = \int_{\mathbb{R}} p(t \cdot v)\, g(v)\, v \, dv$$ ✓

- Taylor $h(v) = v \cdot p(vt) \quad \Rightarrow \quad h(v) \approx h(w) + \sum_{l=1}^{\infty} \frac{d^l h}{dv^l}(w) \frac{(v-w)^l}{l!}$

$$\frac{dh}{dv} = p(vt) + v \frac{dp}{d(vt)} \cdot \frac{d(vt)}{dv} = p(vt) + vt \frac{dp}{d(vt)}$$

$$\frac{d^2 h}{dv^2} = t \cdot \frac{dp}{d(vt)} + t \cdot \frac{dp}{d(vt)} + vt \cdot \frac{d^2 p}{d(vt)^2} \cdot t$$

$$\Rightarrow \frac{d^m h}{dv^m} = m \cdot t^{m-1} \frac{d^{m-1} p}{d(vt)^{m-1}} + v t^m \frac{d^m p}{d(vt)^m}$$ ✓

$$\Rightarrow h(v) \Big|_{\substack{v \approx w \\ w = E[g(v)]}} = w \cdot p(wt) + \sum_{l=1}^{\infty} \frac{(v-w)^l}{l!} \left[m \cdot t^{m-1} \frac{d^{m-1} p}{d(vt)^{m-1}}(w) + w t^m \frac{d^m p}{d(wt)^m}(w) \right]$$ ✓

- $$J(t) = \int_{\mathbb{R}} g(v)\, w \cdot p(wt)\, dv + \int_{\mathbb{R}} g(v) \sum_{l=1}^{\infty} \frac{(v-w)^l}{l!} \cdot [\;]\, dv$$

$$= w\, p(wt) \underbrace{\int_{\mathbb{R}} g(v)\, dv}_{1} + \sum_{l=1}^{\infty} [\;] \cdot \frac{1}{l!} \underbrace{\int_{\mathbb{R}} g(v)(v-w)^l\, dv}_{\sigma \mu_l} \qquad \Big| \; g(v) = N e^{-\frac{(v-w)^2}{2\sigma^2}}$$

$$= w\, p(wt) + \sum_{l=1}^{\infty} \frac{\sigma \mu_l}{l!} [\;] = (*) =$$ ✓

$$= w\, p(wt) + \sum_{k=1}^{\infty} \frac{(2k)!}{(2k)!!} \sigma^{2k} \frac{1}{(2k)!} \left[2k t^{2k-1} \frac{d^{2k-1} p}{d(vt)^{2k-1}}(w) + w t^{2k} \frac{d^{2k} p}{d(vt)^{2k}}(w) \right]$$

$$= w\, p(wt) + \sum_{k=1}^{\infty} \frac{\sigma^{2k}}{(2k)!!} [\;]$$ ✓

(*) →

$$I = \int e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}} \qquad \Big|\ a = \frac{1}{2\sigma^2} \qquad \Big|\ \frac{d}{da}$$

$$+\int x^2 e^{-ax^2}\,dx = \sqrt{\pi}\,(+1)\,\frac{1}{2}\,a^{-\frac{3}{2}} \qquad \Big|\ \frac{d}{da}$$

$N \cdot \mu_2$

$$+\int x^4 e^{-ax^2}\,dx = \sqrt{\pi}\,(+1)\,\frac{1}{2}\cdot\frac{3}{2}\,a^{-\frac{5}{2}}$$

$N \cdot \mu_4$

$$\frac{d^k I}{da^k} \stackrel{(x)}{=} -\sqrt{\pi}\,\frac{1}{2^k}\,(2k-1)!!\,a^{-\frac{2k+1}{2}}$$

$$= -\sqrt{\pi}\,\frac{1}{2^k}\,(2k-1)!!\,(2\sigma^2)^{\frac{2k+1}{2}}$$

$$= -\sqrt{\pi}\,\frac{1}{\cancel{2^k}}\,(2k-1)!!\,\cancel{2^k}\cdot 2^{\frac{1}{2}}\,\sigma^{2k}\cdot\sigma^1$$

$$= -\sqrt{\pi\, 2\sigma^2}\,(2k-1)!!\,\sigma^{2k}$$

$$\mu_{2k} = \underbrace{\frac{1}{\sqrt{2\pi\sigma^2}}}_{\text{NORMALIZACE}} \cdot \underbrace{(-1)}_{\text{ZNAMÉNKO}} \cdot \frac{d^k I}{da^k}$$

$$\Rightarrow \quad \mu_{2k} = (2k-1)!!\,\sigma^{2k} \qquad \Big|\ (2k-1)!! = \frac{(2k)!}{(2k)!!}\ \Big|$$

$$= \frac{(2k)!}{(2k)!!}\,\sigma^{2k}$$

$\mu_{2k-1} = 0$ .. centrální moment symetrické fce

(x) záměny: 1) SPOJITÉ → MĚŘITELNÉ

2) $a = \frac{1}{2}$ → MOMENTY GAUSE S $\sigma^2 = 1$

3) DERIVACÍ MOMENTU GAUSSE DOSTANU KOMBINACI PŘEDCHOZÍCH MOMENTŮ, TY JSOU VŠECHNY OMEZENÉ.

10

$$p(x) = 4x e^{-2x} =: p_0(x)$$

$$p_1(x) = \Theta(x) \int_0^x 4s e^{-2s}\, 4(x-s) e^{-2(x-s)}\, ds = \Theta(x)\, 4^2 e^{-2x} \int_0^x (x \cdot s - s^2)\, ds =$$

$$= \Theta(x)\, 4^2 e^{-2x} \left[x \frac{s^2}{2} - \frac{s^3}{3} \right]_0^x = \Theta(x) \frac{4^2}{6} x^3 e^{-2x}$$

$$p_2(x) = \Theta(x) \int_0^x \frac{4^2}{6} s^3 e^{-2s}\, 4(x-s)\, e^{-2(x-s)}\, ds = \Theta(x) \frac{4^3}{6} e^{-2x} \left[x \frac{s^4}{4} - \frac{s^5}{5} \right]_0^x =$$

$$= \Theta(x) \frac{4^3}{6 \cdot 20} e^{-2x} x^5 e^{-2x}$$

$$p_n(x) = \Theta(x) \frac{4^{n+1}}{(2n+1)!} e^{-2x} x^{2n+1}$$

$$R(x) = \sum_{n=0}^{\infty} \Theta(x) \frac{4^{n+1}}{(2n+1)!} e^{-2x} x^{2n+1} = 2\Theta(x) e^{-2x} \sum_{n=0}^{\infty} \frac{(2x)^{2n+1}}{(2n+1)!!} =$$

$$= 2\,\Theta(x) e^{-2x} \sinh(2x) = 2\,\Theta(x) e^{-2x} \frac{e^{2x} - e^{-2x}}{2} =$$

$$= 2\,\Theta(x) \frac{1 - e^{-4x}}{2} = \Theta(x)\left(1 - e^{-4x}\right)$$

Rozhodneme o SK na $\langle 0, L \rangle$!

$$\left| \frac{4^{n+1}}{(2n+1)!} e^{-2x} x^{2n+1} \right| \leq \frac{4^{n+1}}{(2n+1)!} e^{-2L} L^{2n+1}$$

$$g_n(x) = x^{2n+1} e^{-2x} \Rightarrow g_n'(x) = \left((2n+1) x^{2n} - 2x^{2n+1} \right) e^{-2x} \overset{!}{=} 0$$

$$2n+1 - 2x \overset{!}{=} 0$$

$$2x = 2n+1$$

$\Rightarrow$ stacionární bod se posouvá doprava

$\Rightarrow$ od určitého „n" vypadne z $\langle 0, L \rangle$

$$\Rightarrow \boxed{\cdot}$$

Konverguje číselná řada $\sum \frac{4^{n+1}}{(2n+1)!} e^{-2L} L^{2n+1}$?

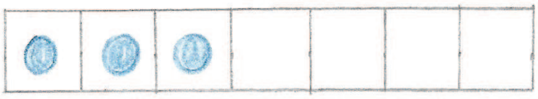
$$\lim_{n\to\infty} \frac{4^{n+2}}{(2n+3)!} \frac{L^{2n+3}}{L^{2n+1}} \frac{(2n+1)!}{4^{n+1}} = \lim_{n\to\infty} \frac{4L^2}{2n+3} \cdot \frac{1}{2n+2} = 0 < 1$$

$\Rightarrow$ konverguje

$\Rightarrow$ podle Weierstrassova kritéria:

$$\sum \frac{4^{n+1}}{(2n+1)!} e^{-2x} x^{2n+1} \overset{\langle 0,L \rangle}{\rightrightarrows}$$

$$\alpha = \frac{1}{2} \wedge \beta = \frac{3}{5} \quad \Rightarrow \quad Z_N \approx \frac{2\beta}{\sqrt{\pi}(2\beta-1)} \frac{4^N}{\sqrt{N}}$$



$$\langle w| E = \frac{1}{\alpha}\langle w| = 2\langle w|$$
$$D|v\rangle = \frac{1}{\beta}|v\rangle = \frac{5}{3}|v\rangle$$

$$P_{ss} = \frac{1}{Z_N}\frac{1}{\langle w|v\rangle}\langle w| D^{n_1} E^{m_1} (D+E)^{l_1} \dots |v\rangle$$

$$n_1 = 3 \wedge m_1 = 0 \wedge l_1 = 0 \wedge n_2 = 0 \wedge m_2 = 4 \wedge l_2 = 0$$

$\langle w|v\rangle = 1$ BÚNO

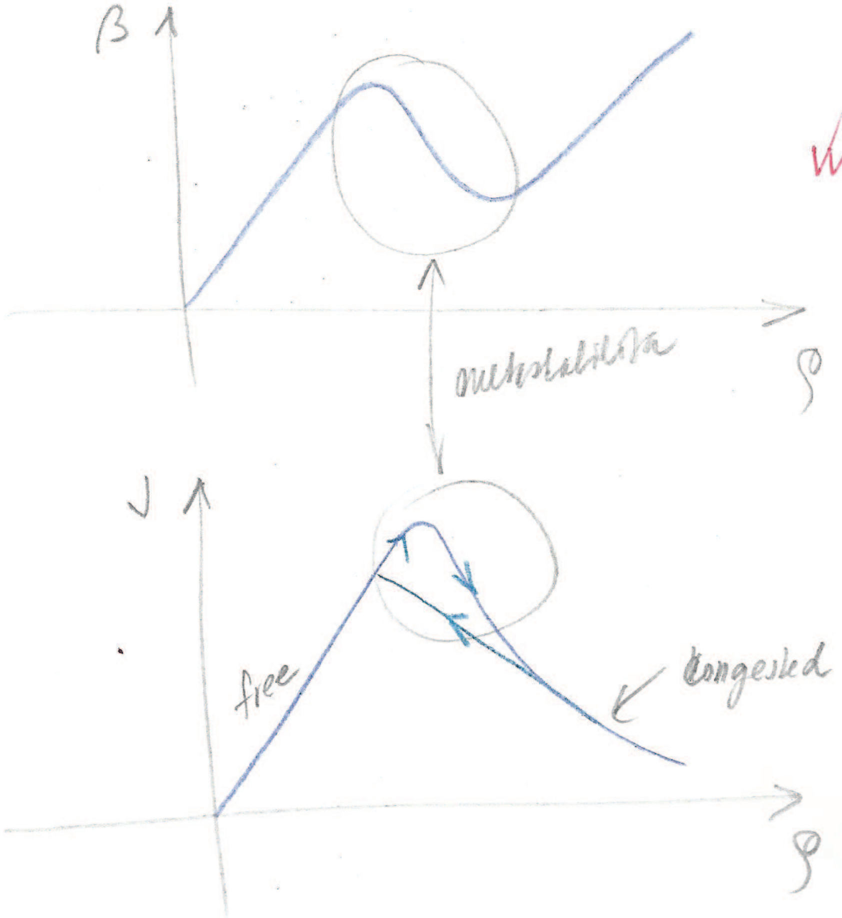
$$\Rightarrow \quad P_{ss} = \frac{1}{Z_N}\langle w| D^3 E^4 v\rangle \quad \wedge \quad Z_N = \frac{\frac{6}{5}}{\frac{1}{5}}\frac{1}{\sqrt{\pi}}\frac{4^7}{\sqrt{7}} = \frac{6 \cdot 4^7}{\sqrt{7\pi}}$$

Výpočet:

$$D^3E^4 = (D+E)^3 E = (D^3 + 3D^2E + 3DE^2 + E^3)E =$$
$$= D^2(D+E) + 3D(D+E)E + 3(D+E)E^2 + E^4 =$$
$$= D^3 + D(D+E) + 3D^2E + 3DE^2 + 3DE^2 + 3E^3 + E^4 =$$
$$= D^3 + D^2 + D + E + 3D(D+E) + 6(D+E)E + 3E^3 + E^4 =$$
$$= D^3 + D^2 + D + E + 3D^2 + 3D + 3E + 6DE + 6E^2 + 3E^3 + E^4 =$$
$$= D^3 + 4D + E + 4D^2 + 3E + 6D + 6E + 6E^2 + 3E^3 + E^4 =$$
$$= D^3 + 4D^2 + 10D + 10E + 6E^2 + 3E^3 + E^4$$

$$\Rightarrow P_{ss} = \frac{\sqrt{7\pi}}{6 \cdot 4^7}\left(\frac{1}{\beta^3} + \frac{4}{\beta^2} + \frac{10}{\beta} + \frac{10}{\alpha} + \frac{6}{\alpha^2} + \frac{3}{\alpha^3} + \frac{1}{\alpha^4}\right)$$

5.



√√

√√

√√ vysvětlení rachy

(6.)

$$P[n=n|L] = \frac{L^n}{n!} e^{-L} \quad 1b$$

8b

$$\langle n(L)\rangle = \sum_{n=0}^{\infty} n \frac{L^n}{n!} e^{-L} = e^{-L} \sum_{n=1}^{\infty} \frac{L^n}{(n-1)!} = L e^{-L} \sum_{m=0}^{\infty} \frac{L^m}{m!} = L \quad 1b$$

$$\langle n^2(L)\rangle = \sum_{n=0}^{\infty} n^2 \frac{L^n}{n!} e^{-L} = e^{-L} \sum_{n=1}^{\infty} n \frac{L^n}{(n-1)!} = s(L)\cdot e^{-L}$$

1b

$$h(L) = \frac{s(L)}{L} = \sum_{n=1}^{\infty} n \frac{L^{n-1}}{(n-1)!}$$

$$H(L) = \sum_{n=1}^{\infty} \frac{L^n}{(n-1)!} = L e^{L} \Rightarrow h(L) = (L+1) e^{L} \quad 1b$$

$$s(L) = L(L+1) e^{+L} \Rightarrow \langle n^2(L)\rangle = L^2 + L \quad 1b$$

$$\Delta L = VAR(n) = L^2 + L - L^2 = \underline{\underline{L}} \quad 1b$$

1b

Řešení přes shlukovou funkci: celé jen za 3 body ($\in$ nedodržení zadání!)

8b

tok v systému:

$$J(x,\tau) = \underbrace{J_e(x,\tau)}_{\text{efektivní tok}} - \underbrace{D\,\frac{\partial \rho(x,\tau)}{\partial x}}_{\text{difúzní porucha } (D>0)}$$

(souvisí se změnami hustoty)

$$\Rightarrow \quad V(x,\tau) = V_e(x,\tau) - \frac{D}{\rho(x,\tau)}\,\frac{\partial \rho(x,\tau)}{\partial x}$$

úpravy rovnice kontinuity:

$$0 = \frac{\partial \rho}{\partial \tau} + \frac{\partial J}{\partial x} = \frac{\partial \rho}{\partial \tau} + \frac{\partial J_e}{\partial x} - D\,\frac{\partial^2 \rho}{\partial x^2} = \frac{\partial \rho}{\partial \tau} + \frac{\partial}{\partial x}\left(\rho \cdot V_e\right) - D\,\frac{\partial^2 \rho}{\partial x^2} =$$

$$= \frac{\partial \rho}{\partial \tau} + \frac{\partial \rho}{\partial x} \cdot V_e + \rho\,\frac{\partial V_e}{\partial x} - D\,\frac{\partial^2 \rho}{\partial x^2} = \Big|$$ Fundam. hypotéza: $V_e(\rho)$ je explic. funkcí pouze ρ $\Big| =$

$$= \frac{\partial \rho}{\partial \tau} + \frac{\partial \rho}{\partial x} V_e + \rho\,\frac{\partial V_e}{\partial \rho}\,\frac{\partial \rho}{\partial x} - D\,\frac{\partial^2 \rho}{\partial x^2} =$$

$$= \frac{\partial \rho}{\partial \tau} + \left(V_e + \rho\,\frac{\partial V_e}{\partial \rho}\right)\frac{\partial \rho}{\partial x} - D\,\frac{\partial^2 \rho}{\partial x^2} \qquad \Big\| \quad V_e(\rho) = V_0\left(1 - \frac{\rho}{\rho_J}\right)$$

Greenshields

$$\frac{\partial \rho}{\partial \tau} + \left(V_0 - V_0\frac{\rho}{\rho_J} + \rho\,\frac{-V_0}{\rho_J}\right)\frac{\partial \rho}{\partial x} - D\,\frac{\partial^2 \rho}{\partial x^2}$$

zavedeme: $\quad u(x,\tau) = V_0 - 2\frac{V_0}{\rho_J}\,\rho(x,\tau) \quad \Rightarrow \quad \rho(x,\tau) = -\frac{\rho_J}{2V_0}\left(u(x,\tau) - V_0\right)$

$$\frac{\partial \rho}{\partial \tau} = -\frac{\rho_J}{2V_0}\,\frac{\partial u}{\partial \tau} \qquad \frac{\partial \rho}{\partial x} = -\frac{\rho_J}{2V_0}\,\frac{\partial u}{\partial x} \qquad \frac{\partial^2 \rho}{\partial x^2} = -\frac{\rho_J}{2V_0}\,\frac{\partial^2 u}{\partial x^2}$$

Dosazení:

$$\frac{\partial u(x,\tau)}{\partial \tau} + u(x,\tau)\,\frac{\partial u(x,\tau)}{\partial x} = D\,\frac{\partial^2 u(x,\tau)}{\partial x^2}$$

10b

$$R(x) = \sum_{n=1}^{\infty} p_n(x) \qquad \langle N_L \rangle = \int_0^L R(x)\,dx$$

$$\langle N_L^2 \rangle = \sum_{n=1}^{\infty} n^2 \mathbb{P}[N_L = n] = \sum_{n=1}^{\infty} n^2 \int_0^L (p_n(x) - p_{n+1}(x))\,dx =$$

$$= \int_0^L \sum_n n^2 p_n(x)\,dx - \int_0^L \sum_n n^2 p_{n+1}(x)\,dx = \left| \begin{matrix} m = n+1 \\ n = m-1 \end{matrix} \right| =$$

$$= \int_0^L \sum_{n=1}^{\infty} n^2 p_n(x)\,dx - \int_0^L \sum_{n=2}^{\infty} (n-1)^2 p_n(x)\,dx = \int_0^L p_1(x)\,dx + \int_0^L \sum_{n=2}^{\infty} (2n-1) p_n(x)\,dx =$$

$$= \int_0^L p_1(x)\,dx + 2\int_0^L \sum_{n=2}^{\infty} n p_n(x)\,dx - \int_0^L \sum_{n=2}^{\infty} p_n(x)\,dx =$$

$$= 2\int_0^L \underline{\sum_n n p_n(x)}\,dx - \int_0^L \sum_{n=1}^{\infty} p_n(x)\,dx = 2\int_0^L \underline{\varphi(x)}\,dx - \int_0^L R(x)\,dx$$

$$\Delta(L) = 2\int_0^L \varphi(x)\,dx - \int_0^L R(x)\,dx - \left(\int_0^L R(x)\,dx\right)^2$$

$$R(x) * R(x) = \left(\sum_n p_n(x)\right) * \left(\sum_n p_n(x)\right) = \int_0^x \sum_{k=1}^{\infty} \sum_{l=1}^{k} p_l(s)\, p_{k-l+1}(x-s)\,ds =$$

$$= \sum_{k=1}^{\infty} \sum_{l=1}^{k} p_l(x) * p_{k-l+1}(x) = \sum_{k=1}^{\infty} \sum_{l=1}^{k} p_{k+1}(x) = \sum_{k=1}^{\infty} k p_{k+1}(x) =$$

$$= \sum_{k=1}^{\infty} (k+1) p_{k+1}(x) - \sum_{k=1}^{\infty} p_{k+1}(x) + p_1(x) - p_1(x) = \sum_{k=1}^{\infty} k p_k(x) - \sum_{k=1}^{\infty} p_k(x) =$$

$$= \underline{\varphi(x) - R(x)} \quad \Rightarrow \quad \varphi(x) = (R*R)(x) + R(x)$$

$\Rightarrow$

$$\Delta(L) = 2\int_0^L (R*R)(x)\,dx + 2\int_0^L R(x)\,dx - \int_0^L R(x)\,dx - \left(\int_0^L R(x)\,dx\right)^2 =$$

$$= 2\int_0^L (R*R)(x)\,dx + \int_0^L R(x)\,dx \left(1 - \int_0^L R(x)\,dx\right)$$

$$x^2y'' + xy' - (x^2+1)y = 0 \qquad z(x) = xe^{x}y(x)$$ 6f

$$\Rightarrow \quad y = \frac{z(x)}{x}e^{-x} \quad \Rightarrow \quad y' = \frac{z'}{x}e^{-x} - \frac{z}{x^2}e^{-x} - \frac{z}{x}e^{-x}$$

$$y'' = \frac{z''}{x}e^{-x} - 2\frac{z'}{x^2}e^{-x} - 2\frac{z'}{x}e^{-x} + 2\frac{z}{x^3}e^{-x} + \frac{z}{x^2}e^{-x} + \frac{z}{x^2}e^{-x} + \frac{z}{x}e^{-x}$$

Dosazení:

$$xz'' - 2z' - 2xz' + 2\frac{z}{x} + 2z + xz + z' - \frac{z}{x} - z - (x^2+1)\frac{z}{x} = 0$$

$$xz'' + z'(-2-2x+1) + z\left(\cancel{\frac{2}{x}} + 2 + \cancel{x} - \cancel{\frac{1}{x}} - 1 - \cancel{\frac{x^2+1}{x}}\right) = 0$$

$$\underline{xz'' - (1+2x)z' + z = 0}$$

aproximace pro malá x:

$$\left.\begin{aligned} z(0) &= 1 \\ z'(0) &= 1 \\ x\cdot z''\Big|_{x=0} &= 0 \end{aligned}\right\} \Rightarrow \quad x\cdot z'' \approx 0$$

$$(1+2x)z' - z = 0$$

$$z' - \frac{1}{1+2x}z = 0 \quad \Big/ \; e^{-\frac{1}{2}\ln(1+2x)} = \frac{1}{\sqrt{1+2x}}$$

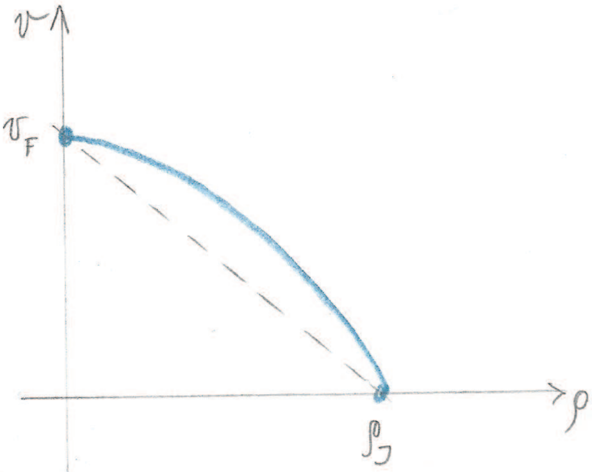
$$\left(z\cdot\frac{1}{\sqrt{1+2x}}\right)' = C$$

$$z(x) = \underbrace{C\cdot\sqrt{1+2x}} \quad \wedge \quad z(0) = 1 \quad \Rightarrow \quad C = 1$$

$$\Rightarrow$$

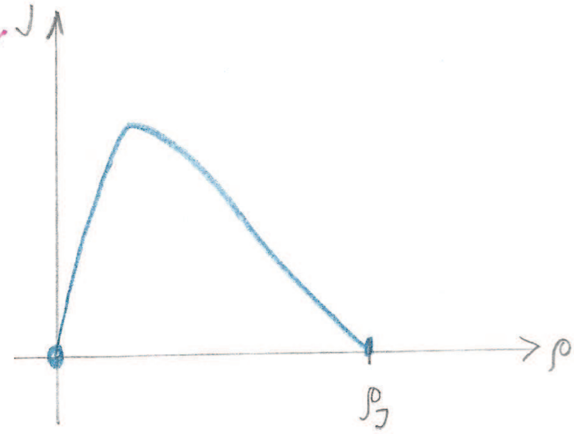
$$\underline{\mathcal{K}_1(x) \approx \frac{z(x)}{x}e^{-x} = \frac{\sqrt{1+2x}}{x}e^{-x}}$$

$$v = v_F\left(1-\frac{\rho}{\rho_J}\right)^3$$



6b

$$J = \rho v = v_F \rho\left(1-\frac{\rho}{\rho_J}\right)^3$$



$$\frac{\partial J}{\partial \rho} = v_F\left(1-\frac{\rho}{\rho_J}\right)^3 - 3v_F\rho\left(1-\frac{\rho}{\rho_J}\right)^2\cdot\frac{1}{\rho_J} \overset{!}{=} 0$$

$$1-\frac{\rho}{\rho_J} - 3\frac{\rho}{\rho_J} = 0$$

$$4\frac{\rho}{\rho_J} = 1 \quad\Rightarrow\quad \rho = \frac{1}{4}\rho_J = \frac{80}{4}\,km^{-1} = 20\,km^{-1}$$

$$J_{max} = J(20) = 128\cdot 20\left(1-\frac{20}{80}\right)^3 = 128\cdot 20\left(\frac{3}{4}\right)^3 = 64\cdot\frac{27}{16\cdot 4}\cdot 40\,h^{-1} =$$

$$= 1080\,h^{-1}$$

Za 10 minut tedy projede $\frac{1}{6}\cdot 1080 = 180$ automobilů

8b

$$\alpha = \frac{1}{2} \wedge \beta = \frac{3}{5} \quad \Rightarrow \quad Z_N \approx \frac{2\beta}{\sqrt{\pi}\,(2\beta-1)} \frac{4^N}{\sqrt{N}}$$

$$\langle w| E = \frac{1}{\alpha}\langle w| = 2\langle w|$$

$$D|v\rangle = \frac{1}{\beta}|v\rangle = \frac{5}{3}|v\rangle$$

$$Z_N = \frac{2\cdot\frac{3}{5}}{\sqrt{\pi}} \frac{1}{\frac{6}{5}-1} \frac{4^8}{\sqrt{8}} = \frac{6}{\sqrt{\pi}} \frac{4^8}{2\sqrt{2}} = \frac{3\cdot 4^8}{\sqrt{2\pi}}$$

$$\mathbb{P}_{SS} = \frac{1}{Z_N}\langle w| EEEDEDDD|v\rangle = \frac{1}{Z_N}\langle w| E^3 D E D^3|v\rangle =$$

$$= \left| \; DE = D+E \; \right| = \frac{1}{Z_N}\langle w| E^3 (D+E) D^3 v\rangle =$$

$$= \frac{1}{Z_N}\cdot 2^3 \cdot \left(\frac{5}{3}\right)^3 \langle w| D+E|v\rangle =$$

$$= \frac{1}{Z_N}\left(\frac{10}{3}\right)^3 \langle w|D|v\rangle + \frac{1}{Z_N}\left(\frac{10}{3}\right)^3 \langle w|E|v\rangle =$$

$$= \frac{1}{Z_N}\left(\frac{10}{3}\right)^3 \left(\frac{5}{3}+2\right) = \frac{1}{Z_N}\left(\frac{10}{3}\right)^3 \frac{11}{3}$$

$$\mathbb{P}_{SS} = \frac{\sqrt{2\pi}}{3\cdot 4^8}\frac{11\cdot 10^3}{3^4} = \frac{\sqrt{2\pi}}{4^8}\frac{11\cdot 10^3}{3^5} = \sqrt{2\pi}\,\frac{11\cdot 10^3}{4^8\cdot 3^5} \approx 0{,}17\%$$

10b

Dyson's gas with logarithmical repulsion 1

$$\mathcal{H} = \sum_{k=1}^{N} \varphi(r_k) = -\sum_{k=1}^{N} \ln(r_k)$$

$$p(\vec{r}) = \frac{1}{Z_N} e^{-\beta \mathcal{H}} = \frac{1}{Z_N} e^{+\beta \sum_{k=1}^{N} \ln(r_k)} = \frac{1}{Z_N} \prod_{k=1}^{N} r_k^{\beta}$$

- systém na kruhu o obvodu L

$$\Rightarrow p(\vec{r}) = \frac{1}{Z_N} \prod_{k=1}^{N} r_k^{\beta} \cdot \delta\Big(L - \sum_k r_k\Big)$$

$$\Rightarrow Z_N(L) = \int_{\mathbb{R}^N} \delta\Big(L - \sum_k r_k\Big) \cdot \prod_{k=1}^{N} r_k^{\beta}\, d\vec{r}$$

$$p(r) = \int_{\mathbb{R}^{N-1}} \frac{1}{Z_N(L)} r^{\beta} \cdot \delta\Big(L - r - \sum_{k=2}^{N} r_k\Big) \cdot \prod_{k=2}^{N} r_k^{\beta} = \frac{Z_{N-1}(L-r)}{Z_N(L)} \cdot r^{\beta}$$

$$\mathcal{L}[Z_N(L)] = \int_0^{\infty}\int_{\mathbb{R}^N} \delta\Big(L - \sum_k r_k\Big) \cdot \prod_k r_k^{\beta}\, d\vec{r}\, dL = \Big| \mathcal{L}[\delta(x-\mu)] = e^{-\mu p} \Big| =$$

$$= \int_{\mathbb{R}^N} \Big(\prod_{k=1}^{N} r_k^{\beta}\Big) \cdot e^{-p \sum_k r_k}\, d\vec{r} = \Big(\int_{\mathbb{R}} r^{\beta} e^{-rp} dr\Big)^N = \big(\mathcal{L}[r^{\beta}]\big)^N =$$

$$= \Big(\frac{\beta!}{p^{\beta+1}}\Big)^N = \frac{\Gamma^N(\beta+1)}{p^{N\beta+N}}$$

$$\Rightarrow Z_N(L) = \mathcal{L}^{-1}\Big[\frac{\Gamma^N(\beta+1)}{\Gamma(N\beta+N)} \cdot \frac{\Gamma(N\beta+N)}{p^{N\beta+N}}\Big] = \frac{\Gamma^N(\beta+1)}{\Gamma(N\beta+N)} L^{N\beta+N-1}$$

$$\Uparrow \quad \mathcal{L}[\Theta(x) x^{\alpha}] = \frac{\Gamma(\alpha+1)}{p^{\alpha+1}}$$

Dyson's gas with logarithmical repulsion 2

$$p(r) = \frac{Z_{N-1}(L-r)}{Z_N(L)} r^{\beta}$$ 1 ∧ vše se odehrává na „jednotkové" kružnici, kdy $L=N$

$$\Rightarrow p(r) = \frac{\Gamma^{N-1}(\beta+1)}{\Gamma(N\beta-\beta+N-1)} \cdot \frac{\Gamma(N\beta+N)}{\Gamma^N(\beta+1)} \frac{(L-r)^{N\beta-\beta+N-2}}{L^{N\beta+N-1}} \cdot r^{\beta} =$$

$$= \frac{r^{\beta}}{\Gamma(\beta+1)} \cdot \frac{\Gamma(N\beta+N)}{\Gamma(N\beta+N-\beta-1)} L^{-\beta-1} \Big(1 - \frac{r}{L}\Big)^{N\beta-\beta+N-2} =$$

$$= \frac{r^{\beta}}{\Gamma(\beta+1)} \frac{(N\beta+N-1)(N\beta+N-2)\dots(N\beta+N-\beta-1)}{L^{\beta+1}} \Big(1 - \frac{r}{L}\Big)^{-\beta-2} \Big(1 - \frac{r}{L}\Big)^{N(\beta+1)} =$$

$$= \left| \begin{matrix} z \cdot \Gamma(z) = \Gamma(z+1) \\ L = N \,! \end{matrix} \right| = \left| \begin{matrix} \text{čitatel nejdelšího zlomku} \\ \text{má } \beta+1 \text{ sečtenců } (\beta \in \mathbb{N}) \end{matrix} \right| =$$

$$= \frac{r^{\beta}}{\Gamma(\beta+1)} \Big(\beta+1-\frac{1}{N}\Big)\Big(\beta+1-\frac{2}{N}\Big)\dots\Big(\beta+1-\frac{\beta+1}{N}\Big)\Big(1-\frac{r}{N}\Big)^{-\beta-2}\Big(1-\frac{r}{N}\Big)^{N(\beta+1)} =$$

$$=: p(r|N)$$

$$p(r) = \lim_{N\to\infty} p(r|N) = \frac{r^{\beta}}{\Gamma(\beta+1)} (\beta+1)^{\beta+1} \cdot 1 \cdot \lim_{N\to\infty} \Big(1 - \frac{r}{N}\Big)^{N(\beta+1)} \Rightarrow$$

$$\rho(r) = \Theta(r) \frac{(\beta+1)^{\beta+1}}{\Gamma(\beta+1)} r^{\beta} e^{-(\beta+1)r}$$