$$\left(\frac{(k+1)!}{2^k}\right)_{k=0}^{\infty} \Rightarrow \quad g(x) \text{ je normovaná } (\mu_0 = 1) \text{ a navíc}$$

$$\mathbb{E}(X) = \mu_1 = \frac{2}{2} = 1 \quad \& \quad \mathbb{E}(X^2) = \frac{3!}{2^2} = \frac{3\cdot 2}{2\cdot 2} = \frac{3}{2}$$

$$\mathbb{V}(X) = \mu_2 - \mu_1^2 = \frac{3}{2} - 1 = \frac{1}{2}$$

$$h(x) = \frac{9}{4} g(x) e^{-x}$$

$$\varkappa_k = \int_{\mathbb{R}} x^k h(x)\,dx = \int_{\mathbb{R}} x^k \frac{9}{4} g(x) e^{-x}\,dx = \frac{9}{4}\int_{\mathbb{R}} x^k g(x) \sum_{n=0}^{\infty} \frac{(-x)^n}{n!}\,dx =$$

$$= \frac{9}{4}\sum_{n=0}^{\infty} \int_{\mathbb{R}} (-1)^n \cdot \frac{1}{n!} x^{k+n} g(x)\,dx = \frac{9}{4}\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}\mu_{k+n}$$

$$\varkappa_k = \frac{9}{4}\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}\frac{(n+k+1)!}{2^{k+n}} = \underline{\frac{9}{4}\left(\frac{1}{2}\right)^k \sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n \frac{(n+k+1)!}{n!}}$$

$$\varkappa_0 = \frac{9}{4}\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n \cdot (n+1) = \frac{9}{4}\,\frac{1}{(3/2)^2} = 1$$

$$\varkappa_1 = \frac{9}{4}\frac{1}{2}\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n (n+1)(n+2) = \frac{9}{4}\frac{1}{2}\frac{2}{(3/2)^3} = \frac{9}{2}\cdot\frac{8}{9\cdot 3}\frac{1}{2} = \frac{2}{3}$$

$$\varkappa_2 = \frac{9}{4}\cdot\frac{1}{4}\sum_{n=0}^{\infty}\left(-\frac{1}{2}\right)^n (n+1)(n+2)(n+3) = \frac{9}{4}\cdot\frac{1}{4}\cdot\frac{6}{(3/2)^4} = \frac{9}{4}\cdot\frac{3}{2}\cdot\frac{16}{9\cdot 9} = \frac{2}{3}$$

$$\mathbb{V}(Y) = \varkappa_2 - \varkappa_1^2 = \frac{2}{3} - \frac{4}{9} = \frac{6}{9} - \frac{4}{9} = \frac{2}{9}$$

$X, Y$ konvolučně sdružené $\Rightarrow$

$$\underline{\mathbb{V}(X+Y) = \mathbb{V}(X) + \mathbb{V}(Y) = \frac{1}{2} + \frac{2}{9} = \frac{9+4}{18} = \frac{13}{18}}$$

Trendová funkce $\lambda(L)$ je de facto střední hodnota intervalových frekvencí, tj.

$$\lambda(L) = \mathbb{E}(\eta_L) \equiv \sum_{k=0}^{\infty} k \cdot \mathbb{P}[\eta_L = k]$$

Shluková funkce $r(x)$ je nekonečným součtem hustot pro multirozteče, tj.

$$r(x) = \sum_{k=0}^{\infty} g_k(x), \quad \text{kde} \quad g_k(x) = \mathop{\ast}_{k} h(x)$$

Ukážeme, že řada $\sum_k g_k(x)$ konverguje stejnoměrně na každém $\langle 0, L \rangle$, což mimo jiné implikuje, že $\sum_k g_k(x)$ konverguje $\forall x \in \mathbb{R}$ a také, že $\sum_k [g_{k-1}(x) - g_k(x)]$ SK

- $|g_0(x)| = h(x) < K$ (generátor je omezená funkce)
- $g_1(x) = (h * h)(x) = \int_0^x h(y) h(x-y)\, dy \leq K^2 \cdot x$
- $g_2(x) = (g_1 * h)(x) = \int_0^x g_1(y) h(x-y)\, dy \leq K^3 \cdot \frac{x^2}{2}$
- $g_k(x) \leq K^{k+1} \frac{x^k}{k!} \underset{\langle 0,L \rangle}{\leq} K^{k+1} \frac{L^k}{k!}$

Řada $\sum_k K^{k+1} \frac{L^k}{k!}$ konverguje jako číselná $\Rightarrow$ (Weierstrass) $\sum_k g_k(x) \overset{\langle 0,L \rangle}{\rightrightarrows}$

Výpočet:

$$\lambda(L) = \sum_{k=1}^{\infty} k \cdot \mathbb{P}[\eta_L = k] = \sum_{k=1}^{\infty} k \left( \int_0^L g_{k-1}(x)\, dx - \int_0^L g_k(x)\, dx \right) =$$

$$= \sum_{k=1}^{\infty} k \int_0^L (g_{k-1}(x) - g_k(x))\, dx = \left| \begin{matrix} \text{příslušná řada konverguje} \\ \text{stejnoměrně na } \langle 0,L \rangle \end{matrix} \right\} \Rightarrow \begin{matrix} \text{záměna} \\ \text{možná} \end{matrix} \Bigg| =$$

$$= \int_0^L \sum_{k=1}^{\infty} k \cdot (g_{k-1}(x) - g_k(x))\, dx = \int_0^L \lim_{n\to\infty} \sum_{k=1}^{n} k \cdot (g_{k-1}(x) - g_k(x))\, dx =$$

$$= \int_0^L \lim_{n\to\infty} \overbrace{\left( \sum_{k=0}^{n-1} g_k(x) + n \cdot g_n(x) \right)}^{(*)} dx = \int_0^L \left( \sum_{k=0}^{\infty} g_k(x) - \lim_{n\to\infty} n \cdot g_n(x) \right) dx =$$

$$= \left| \begin{matrix} \lim_{n\to\infty} n \cdot g_n(x) = 0 \Leftarrow \text{z nutné podmínky} \\ \text{pro konvergenci řady } \sum k \cdot g_k(x) \end{matrix} \right| = \int_0^L \sum_{k=0}^{\infty} g_k(x)\, dx = \int_0^L r(x)\, dx$$

(*) $$\sum_{k=1}^{n} k(g_{k-1} - g_k) = g_0 \overbrace{- g_1 + 2g_1} \overbrace{- 2g_2 + 3g_2} - 3g_3 + \dots \overbrace{+ n g_{n-1}} \mp n \cdot g_n =$$

$$= g_0 + g_1 + g_2 + g_3 + \dots + g_{n-1} - n \cdot g_n$$

$$A \cdot x^2 \cdot e^{-\frac{\beta}{x^2}} e^{-\lambda x} * A \cdot x^2 \cdot e^{-\frac{\beta}{x^2}} e^{-\lambda x} =$$

$$= \Theta(x) \cdot A^2 \int_0^x s^2 e^{-\frac{\beta}{s^2}} e^{-\lambda s} (x-s)^2 \cdot e^{-\frac{\beta}{(x-s)^2}} e^{-\lambda(x-s)} ds =$$

$$= \Theta(x) \cdot A^2 \cdot e^{-\lambda x} \int_0^x s^2 (x-s)^2 e^{-\beta\left[\frac{1}{s^2} + \frac{1}{(x-s)^2}\right]} ds = (*)$$

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- chceme aplikovat metodu hrubého leadingu a pro ni potřebujeme najít maximum funkce $-\frac{1}{s^2} - \frac{1}{(x-s)^2}$, tj. minimum funkce

$$h(s) = \frac{1}{s^2} + \frac{1}{(x-s)^2} \quad \text{na } \langle 0, x \rangle$$

- přitom $\lim_{s \to 0_+} h(s) = +\infty \wedge \lim_{s \to x_-} h(x) = +\infty \wedge h(s) \in C^1(0,x)$
- $h'(s) = -\frac{2}{s^3} + \frac{2}{(x-s)^3} \overset{!}{=} 0 \Rightarrow s^3 = (x-s)^3 \Rightarrow s = \frac{x}{2}$
- $s_0 = \frac{x}{2}$ je zjevně bodem minima
- funkční hodnota v minimu: $h\left(\frac{x}{2}\right) = \frac{4}{x^2} + \frac{4}{x^2} = \frac{8}{x^2}$

$$(*) \doteq \Theta(x) \cdot A^2 \cdot e^{-\lambda x} \int_0^x s^2 (x-s)^2 \cdot e^{-\frac{8\beta}{x^2}} ds =$$

$$= \Theta(x) A^2 e^{-\lambda x} \int_0^x s^2 (x-s)^2 ds \cdot e^{-\frac{8\beta}{x^2}} = \left| \begin{array}{l} s = u \cdot x \\ ds = x\, du \end{array} \right| =$$

$$= \Theta(x) A^2 e^{-\lambda x} e^{-\frac{8\beta}{x^2}} \int_0^1 u^2 \cdot x^2 \cdot (x - x \cdot u)^2 x \cdot du =$$

$$= \Theta(x) \cdot A^2 \cdot e^{-\frac{8\beta}{x^2}} e^{-\lambda x} x^5 \int_0^1 u^2 (1-u)^2 du = \underline{\Theta(x) \cdot \frac{A^2}{30} x^5 \cdot e^{-\frac{8\beta}{x^2}} e^{-\lambda x}}$$

Tvrzení:

$g(x), h(x) \in \mathcal{B}$ & $G(s) = \mathcal{L}[g] \wedge H(s) = \mathcal{L}[h]$

Pak platí:
$$\int_0^\infty g(x) H(x)\, dx = \int_0^\infty G(x) h(x)\, dx$$

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Důkaz:

$G(s), H(s)$ jsou omezené na $\langle 0; +\infty)$

$\Leftarrow G(0) = \mu_0 \wedge G(s) \in C^\infty(\mathbb{R}^+) \wedge G'(s) < 0$ na $\langle 0; +\infty) \wedge$

$\wedge\ G(s)$ nezáporná $\wedge \lim_{s \to +\infty} G(s) = 0$

Proto:
$$|G(x) h(x)| \leqq K \cdot h(x) \in \mathcal{L}(0; +\infty)$$
$$\Rightarrow G(x) h(x) \in \mathcal{L}(0; +\infty)$$

Analogicky: $H(x) g(x) \in \mathcal{L}(0; +\infty)$

Prokázání rovnosti:
$$\int_0^\infty g(x) H(x)\, dx = \int_0^\infty g(x) \int_{\mathbb{R}} h(y) \cdot e^{-xy}\, dy\, dx = \left| \text{Fubiny č. 1} \right| =$$
$$= \int_{\mathbb{R}^2} \Theta(x) g(x) \Theta(y) h(y) e^{-xy}\, d(x,y) = \left| \text{Fubiny č. 2} \right| =$$
$$= \int_{\mathbb{R}} \Theta(y) h(y) \left[ \int_{\mathbb{R}} g(x) \Theta(x) e^{-xy} dx \right] dy = \int_{\mathbb{R}} \Theta(y) h(y) G(y)\, dy =$$
$$= \int_0^{+\infty} h(y) G(y)\, dy$$

$\mathcal{L}[y] = Y \quad \mathcal{L}[w] = W$

$\mathcal{L}[y'] = sY - 1 \quad \mathcal{L}[y''] = s \cdot \mathcal{L}[y'] + 1 = s^2 Y - s + 1$

Dosazení do II. rovnice:

$$s^2 Y - s + 1 + 2sY - 2 + 5Y = 0$$

$$Y(s^2 + 2s + 5) = 1 + s$$

$$Y = \frac{1+s}{s^2+2s+5}$$

$$y(x) = \mathcal{L}^{-1}\left[\frac{(1+s)}{(s+1)^2+4}\right] = \underline{\underline{\Theta(x) \cdot e^{-x} \cos(2x)}}$$

První rovnice:

$$\mathcal{L}[e^{-x} \cos x \cdot \sin x] = \frac{1}{2} \mathcal{L}[e^{-x} \sin(2x)] = \frac{1}{(s+1)^2+4} = \frac{1}{s^2+2s+5}$$

$$-\mathcal{L}[x \cdot y(x)] = \frac{dY}{ds} = \frac{s^2+2s+5-(s+1)(2s+2)}{(s^2+2s+5)^2} = \frac{s^2+2s+5-2s^2-4s-2}{(s^2+2s+5)^2}$$

$$\mathcal{L}[x \cdot y(x)] = \frac{-s^2-2s+3}{(s^2+2s+5)^2}(-1)$$

Dosazení do I. rovnice:

$$W^2 = \frac{1}{s^2+2s+5} + \frac{-s^2-2s+3}{(s^2+2s+5)^2}$$

$$W^2 = \frac{8}{(s^2+2s+5)^2}$$

$$W = \frac{2\sqrt{2}}{s^2+2s+5} = \frac{2\sqrt{2}}{(s+1)^2+4}$$

$$\underline{\underline{w(x) = \sqrt{2}\, \Theta(x) \cdot e^{-x} \sin(2x)}}$$

$$\int_0^\infty x^n e^{-\lambda x}\,dx = \frac{n!}{\lambda^{n+1}}$$

$$\left.\begin{aligned}\int_0^\infty h(x)\,dx &= A\int_0^\infty x^2 e^{-\lambda x}\,dx = A\cdot\frac{2}{\lambda^3} \overset{!}{=} 1\\ \int_0^\infty x\,h(x)\,dx &= A\int_0^\infty x^3 e^{-\lambda x}\,dx = A\cdot\frac{6}{\lambda^4} \overset{!}{=} 1\end{aligned}\right\} \Rightarrow \frac{\lambda^4}{6}\cdot\frac{2}{\lambda^3} \overset{!}{=} 1 \Rightarrow \lambda = 3$$

$$\Rightarrow A = \frac{3^3}{2}$$

$$\underline{h(x) = \frac{27}{2}\,\Theta(x)\cdot x^2\cdot e^{-3x}}$$

$$H(s) = \frac{27}{2}\,\frac{2}{(s+3)^3} = \frac{27}{(s+3)^3}$$

$$R(s) = \frac{H(s)}{1-H(s)} = \frac{27}{(s+3)^3}\cdot\frac{(s+3)^3}{(s+3)^3-27} = \frac{27}{(s+3)^3-27} = \frac{27}{s^3+9s^2+27s}$$

$$R(s) = \frac{A}{s} + \frac{Bs+C}{s^2+9s+27} \Rightarrow (A,B,C) = (1,-1,-9)$$

$$R(s) = \frac{1}{s} - \frac{s+9}{\left(s+\frac{9}{2}\right)^2+\frac{27}{4}} = \frac{1}{s} - \frac{s+9/2}{\left(s+\frac{9}{2}\right)^2+\frac{27}{4}} - \frac{9/2}{\left(s+\frac{9}{2}\right)^2+\frac{27}{4}}$$

$$\underline{r(x) = \Theta(x)\cdot\left[1 - e^{-\frac{9x}{2}}\cos\left(\frac{3\sqrt{3}}{2}x\right) - \sqrt{3}\,e^{-\frac{9x}{2}}\sin\left(\frac{3\sqrt{3}}{2}x\right)\right]}$$