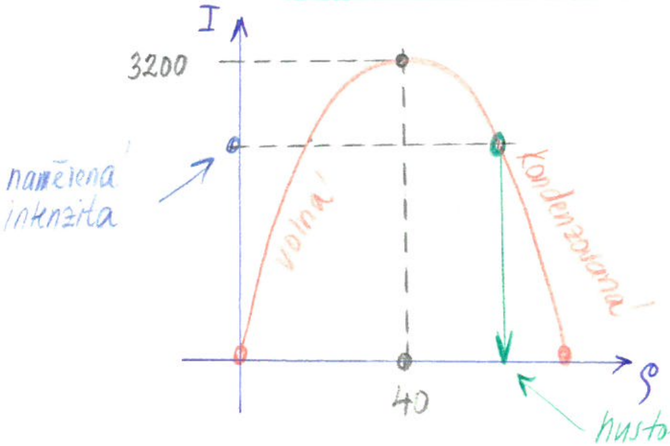
měření trvalo 3 minuty, tj. $\frac{3}{60} = \frac{1}{20}$ hodiny

za tu dobu projelo danou oblastí 120 🚗 $\Rightarrow$ intenzita $I = \frac{120}{1/20} = 2400$ 🚗/h

Určení přesného tvaru fundamentální křivky: $I = -\alpha(\rho - 40)^2 + \beta \qquad (\alpha, \beta) = ?$

$$\rho = 40 \Rightarrow I = 3200 \Rightarrow \beta = 3200$$
$$\rho = 0 \Rightarrow I = 0 \Rightarrow 0 = -\alpha \cdot 1600 + 3200 \Rightarrow \alpha = 2$$

$$I = -2(\rho - 40)^2 + 3200$$



hustota provozu během měření: $\rho = 60$ 🚗/km

$$2400 = -2(\rho - 40)^2 + 3200$$
$$400 = (\rho - 40)^2$$
$$20 = |\rho - 40|$$
$$\rho = \begin{cases} 20 \ldots \text{volná doprava} \\ 60 \ldots \text{kondenzovaná doprava} \end{cases}$$

Při hustotě $\rho = 60$ je podle obrázku: $\beta \doteq 3$

$$g(r) = \frac{(\beta+1)^{\beta+1}}{\Gamma(\beta+1)} r^{\beta} e^{-(\beta+1)r} = \frac{4^4}{3!} r^3 e^{-4r}$$

$$\mathcal{L}[g(r)] = \frac{(\beta+1)^{\beta+1}}{\Gamma(\beta+1)} \frac{\Gamma(\beta+1)}{(s+\beta+1)^{\beta+1}} = \left(\frac{\beta+1}{s+\beta+1}\right)^{\beta+1} \triangleq G(s)$$

$$\beta = 3 \Rightarrow G(s) = \left(\frac{4}{s+4}\right)^4 \qquad G(0) = 1 \checkmark$$

$$G'(s) = 4^4(-4)\cdot(s+4)^{-5}$$
$$G''(s) = 4^5 \cdot 5 (s+4)^{-6} \Rightarrow G''(0) = 4^5 \cdot 5 \cdot \frac{1}{4^6} = \frac{5}{4}$$

$\Rightarrow VAR(R) = \frac{5}{4} - 1 = \frac{1}{4}$ ... tohle je ale rozptyl škálovaných světlostí prostorových rozestupů

$\Rightarrow SD(R) = \frac{1}{2}$

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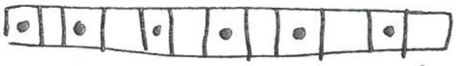
Rozptyl reálných rozestupů:

$$VAR(R) \cdot E^2(R) = \frac{VAR(R)}{\rho^2} = \frac{1}{4} \cdot \left(\frac{1000m}{60}\right)^2 = \frac{2500}{36} m^2 \doteq 70\, m^2$$

Směrodatná odchylka:

$$SD(R) \cdot E(R) = \frac{SD(R)}{\rho} = \frac{1}{2} \cdot \frac{1000m}{60} = \frac{50}{6} m = \frac{25}{3} m \doteq 8m$$

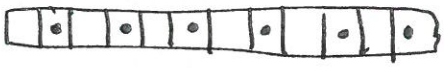
(3) $N = 12, \quad \alpha, \beta = 1/3 \Rightarrow$ není na linii komutativity

1) 

$$P_1 = \frac{1}{Z_{12}} \frac{\langle w|(DE)^6|v\rangle}{\langle w|v\rangle} = \frac{1}{Z_{12}} \frac{\langle w|(D+E)^6|v\rangle}{\langle w|v\rangle} = \frac{1}{Z_{12}} \cdot Z_6$$

$$Z_N \approx \frac{(1-2\alpha)^2}{(1-\alpha)^2} \cdot \frac{N}{\alpha^N (1-\alpha)^N}$$

$$P_1 \approx \cancel{\frac{(1-2\alpha)^2}{(1-\alpha)^2}} \frac{6}{\alpha^6(1-\alpha)^6} \cdot \cancel{\frac{(1-\alpha)^2}{(1-2\alpha)^2}} \cdot \frac{\alpha^{12}(1-\alpha)^{12}}{12} = \frac{\alpha^6(1-\alpha)^6}{2} = \left(\frac{1}{3}\right)^6 \cdot \left(\frac{2}{3}\right)^6 \cdot \frac{1}{2} = \frac{2^5}{3^{12}}$$

2) 

$$P_2 = \frac{1}{Z_{12}} \frac{\langle w|(ED)^6|v\rangle}{\langle w|v\rangle} = \frac{1}{Z_{12}} \frac{\langle w|EDEDEDEDEDED|v\rangle}{\langle w|v\rangle} =$$

$$= \frac{1}{Z_{12}} \cdot \frac{1}{\alpha} \frac{\langle w|DEDEDEDEDE|v\rangle}{\langle w|v\rangle} \cdot \frac{1}{\beta} =$$

$$= \frac{1}{Z_{12}} \cdot \frac{1}{\alpha^2} \frac{\langle w|(DE)^5|v\rangle}{\langle w|v\rangle} = \frac{1}{Z_{12}} \frac{1}{\alpha^2} \frac{\langle w|(D+E)^5|v\rangle}{\langle w|v\rangle} =$$

$$= \frac{1}{Z_{12}} \cdot \frac{1}{\alpha^2} Z_5 \approx \frac{\alpha^{12}(1-\alpha)^{12}}{12} \cdot \cancel{\frac{(1-\alpha)^2}{(1-2\alpha)^2}} \cdot \frac{1}{\alpha^2} \cancel{\frac{(1-2\alpha)^2}{(1-\alpha)^2}} \frac{5}{\alpha^5(1-\alpha)^5} =$$

$$= \frac{\alpha^7(1-\alpha)^7}{12} \cdot \frac{1}{\alpha^2} \cdot 5 = \frac{5}{12} \alpha^5 (1-\alpha)^7 = \frac{5}{12} \cdot \frac{1}{3^5} \left(\frac{2}{3}\right)^7 = \frac{5}{3 \cdot 4} \frac{2^7}{3^{12}} = \frac{5 \cdot 2^5}{3^{13}}$$

$$\frac{P_1}{P_2} = \frac{\alpha^6(1-\alpha)^6}{2} \cdot \frac{12}{5} \frac{1}{\alpha^5(1-\alpha)^7} = \frac{12}{10} \frac{\alpha}{1-\alpha} = \frac{12}{10} \frac{1}{3} \cdot \frac{3}{2} = \frac{12}{20} = \frac{6}{10}$$

$6P_2 = 10P_1 \Rightarrow P_2 > P_1 \Rightarrow$ s větší pravděpodobností nastává druhá konfigurace generátor $h(x)$ a jeho momenty: $\mu_1(h)=\lambda$; $\mu_2(h) \triangleq \varkappa$

hustoti pro k-tou multivztec:

$$g_k(x) = \overset{k}{\underset{m=0}{*}} h(x)$$

úloha: $\mu_1(g_k)=?$ & $\mu_2(g_k)=?$ & $\nu_2(g_k)=\mu_2(g_k)-\mu_1^2(g_k)=?$

označme:

$$H(s) \triangleq \mathcal{L}[h(x)] = \int_{\mathbb{R}} h(x)\, e^{-sx}\,dx \;\Rightarrow\; H'(0)=-\lambda \;\&\; H''(0)=\varkappa$$

$$G_k(s) \triangleq \mathcal{L}[g_k(x)] = \int_{\mathbb{R}} g_k(x)\, e^{-sx}\,dx \;\Rightarrow\; \mu_1(g_k)=G_k'(0)\cdot(-1)$$

$$\mu_2(g_k)=G_k''(0)$$

z i.i.d vlastnosti a z konvolučního teorému víme:

$$G_k(s)=H^{k+1}(s)$$

$$G_k'(s) = (k+1)\cdot H^k(s)\cdot H'(s)$$

$$G_k''(s) = (k+1)\cdot k\cdot H^{k-1}(s)\cdot (H'(s))^2 + (k+1)\cdot H^k(s)\cdot H''(s)$$

$$G_k'(0) = (k+1)\cdot H^k(0)\cdot H'(0) = -(k+1)\cdot 1^k\cdot \lambda = -(k+1)\lambda$$

$$G_k''(0) = (k+1)\cdot k\cdot 1^{k-1}\cdot(-\lambda)^2 + (k+1)\cdot 1^k\cdot\varkappa = \varkappa + k(k+1)\lambda^2$$

$$\Rightarrow\; \mu_1(g_k) = (k+1)\cdot\lambda \;\&\; \mu_2(g_k) = (k+1)\varkappa + k(k+1)\lambda^2$$

$$\nu_2(g_k) = \mu_2(g_k) - \mu_1^2(g_k) = (k+1)\varkappa + k(k+1)\lambda^2 - (k+1)^2\lambda^2 =$$

$$= (k+1)\varkappa + (k+1)\lambda^2(k-k-1) = (k+1)\varkappa - (k+1)\lambda^2 =$$

$$= (k+1)\cdot(\varkappa-\lambda^2) \;\Rightarrow\; VAR(X_k) = (k+1)\cdot VAR(R_0)$$

Poissonovský systém: $h(x) = \Theta(x)\cdot\lambda^{-1} e^{-\frac{x}{\lambda}} = \frac{\Theta(x)}{\lambda} e^{-\frac{x}{\lambda}} \Rightarrow \mu_1(h)=\lambda$

& $\mu_2(h) = 2\lambda^2 \Rightarrow VAR(R_0) = 2\lambda^2-\lambda^2=\lambda^2$

$$\Rightarrow\; VAR(X_k) = (k+1)\cdot\lambda^2$$

(6) $$\frac{\partial u}{\partial \tau} + u(x,\tau)\frac{\partial u}{\partial x} - D\frac{\partial^2 u}{\partial x^2} = 0$$

Cole-Hopsova transformace: $u = -\frac{2D}{\varphi}\frac{\partial \varphi}{\partial x}$

$$\frac{\partial u}{\partial \tau} = +\frac{2D}{\varphi^2}\frac{\partial \varphi}{\partial \tau}\frac{\partial \varphi}{\partial x} - \frac{2D}{\varphi}\frac{\partial^2 \varphi}{\partial \tau \partial x}$$

$$\frac{\partial u}{\partial x} = \frac{2D}{\varphi^2}\left(\frac{\partial \varphi}{\partial x}\right)^2 - \frac{2D}{\varphi}\frac{\partial^2 \varphi}{\partial x^2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\frac{4D}{\varphi^3}\left(\frac{\partial \varphi}{\partial x}\right)^3 + \frac{2D}{\varphi^2}\cdot 2\left(\frac{\partial \varphi}{\partial x}\right)\frac{\partial^2 \varphi}{\partial x^2} + \frac{2D}{\varphi^2}\frac{\partial \varphi}{\partial x}\frac{\partial^2 \varphi}{\partial x} - \frac{2D}{\varphi}\frac{\partial^3 \varphi}{\partial x^3} =$$

$$= -\frac{4D}{\varphi^3}\left(\frac{\partial \varphi}{\partial x}\right)^3 + \frac{6D}{\varphi^2}\left(\frac{\partial \varphi}{\partial x}\right)\frac{\partial^2 \varphi}{\partial x^2} - \frac{2D}{\varphi}\frac{\partial^3 \varphi}{\partial x^3}$$

$$\frac{2D}{\varphi^2}\frac{\partial \varphi}{\partial \tau}\frac{\partial \varphi}{\partial x} - \frac{2D}{\varphi}\frac{\partial^2 \varphi}{\partial \tau \partial x} + \left(-\frac{2D}{\varphi}\frac{\partial \varphi}{\partial x}\right)\left(\cancel{\frac{2D}{\varphi^2}\left(\frac{\partial \varphi}{\partial x}\right)^2} - \frac{2D}{\varphi}\frac{\partial^2 \varphi}{\partial x^2}\right) +$$

$$+ \cancel{\frac{4D^2}{\varphi^3}\left(\frac{\partial \varphi}{\partial x}\right)^3} - \frac{6D^2}{\varphi^2}\left(\frac{\partial \varphi}{\partial x}\right)\frac{\partial^2 \varphi}{\partial x^2} + \frac{2D^2}{\varphi}\frac{\partial^3 \varphi}{\partial x^3} = 0$$

$$\frac{2D}{\varphi^2}\frac{\partial \varphi}{\partial \tau}\frac{\partial \varphi}{\partial x} - \frac{2D}{\varphi}\frac{\partial^2 \varphi}{\partial \tau \partial x} + \frac{4D^2}{\varphi^2}\frac{\partial \varphi}{\partial x}\left(\frac{\partial^2 \varphi}{\partial x^2}\right) - \frac{6D^2}{\varphi^2}\left(\frac{\partial \varphi}{\partial x}\right)\frac{\partial^2 \varphi}{\partial x} + \frac{2D^2}{\varphi}\frac{\partial^3 \varphi}{\partial x^3} = 0$$

$$\frac{2D}{\varphi^2}\frac{\partial \varphi}{\partial \tau}\frac{\partial \varphi}{\partial x} - \frac{2D}{\varphi}\frac{\partial^2 \varphi}{\partial \tau \partial x} - \frac{2D^2}{\varphi^2}\frac{\partial \varphi}{\partial x}\left(\frac{\partial^2 \varphi}{\partial x^2}\right) + \frac{2D^2}{\varphi}\frac{\partial^3 \varphi}{\partial x^3} = 0 \quad \Big| \cdot \frac{\varphi^2}{2D}$$

$$\frac{\partial \varphi}{\partial \tau}\frac{\partial \varphi}{\partial x} - \varphi\frac{\partial^2 \varphi}{\partial \tau \partial x} - D\frac{\partial \varphi}{\partial x}\left(\frac{\partial^2 \varphi}{\partial x^2}\right) + \varphi D\frac{\partial^3 \varphi}{\partial x^3} = 0$$

$$\frac{\partial \varphi}{\partial x}\left(\frac{\partial \varphi}{\partial \tau} - D\frac{\partial^2 \varphi}{\partial x^2}\right) - \varphi\frac{\partial}{\partial x}\left(\frac{\partial \varphi}{\partial \tau} - D\frac{\partial^2 \varphi}{\partial x^2}\right) = 0$$

$$\frac{\partial \varphi}{\partial x}\left(\frac{\frac{\partial \varphi}{\partial \tau} - D\left(\frac{\partial^2 \varphi}{\partial x^2}\right)}{\varphi}\right) = 0$$

$$\Rightarrow \frac{\partial \varphi}{\partial \tau} - D\left(\frac{\partial^2 \varphi}{\partial x}\right) = \varphi \cdot C, \quad C = \text{konst}$$

~~hyperbolická~~ parabolická ✓ ∗ −1

(4)

$$P(\vec{r}) = \frac{1}{Z_N(L)} e^{-\beta \sum_{k=1}^{N} \varphi(r_k)} \Theta(\vec{r}) \cdot \delta\left(L - \sum_{k=1}^{N} r_k\right)$$

$$g(r_1) = \int_{\mathbb{R}^{N-1}} P(\vec{r})\, d(r_2, r_3, \dots, r_N) = \frac{1}{Z_N(L)} \Theta(r_1) e^{-\beta\varphi(r_1)} \int_{\mathbb{R}^{N-1}} e^{-\beta \sum_{k=2}^{N} \varphi(r_k)} \underbrace{\delta\left(L - r_1 - \sum_{k=2}^{N} r_k\right)}_{\Theta(r_2, r_3, \dots, r_N)} d(r_2, \dots, r_N) =$$

$$= \Theta(r_1) \frac{Z_{N-1}(L-r_1)}{Z_N(L)} e^{-\beta\varphi(r_1)}$$

$$\Rightarrow g(r) = \Theta(r) \frac{Z_{N-1}(L-r)}{Z_N(L)} e^{-\beta\varphi(r)}$$

$$\varphi(r) = -\ln r \quad \Rightarrow \quad g(r) = \Theta(r) \frac{Z_{N-1}(L-r)}{Z_N(L)} r^{\beta}$$

$$\mathcal{L}[Z_N(L)] = \int_0^{\infty} \int_{\mathbb{R}^N} \delta\left(L - \sum_{k=1}^{N} r_k\right) \cdot \prod_{k=1}^{N} \Theta(r_k) \cdot r_k^{\beta} e^{-sL} d\vec{r}\, dL = \left| \mathcal{L}[\delta(x-\mu)] = e^{-\mu s} \right| =$$

$$= \int_{\mathbb{R}^N} \prod_{k=1}^{N} \Theta(r_k) r_k^{\beta} e^{-s \sum_{k=1}^{N} r_k} d\vec{r} = \left(\int_0^{+\infty} \Theta(r) \cdot r^{\beta} e^{-sr} dr \right)^N = \mathcal{L}^N[\Theta(r) \cdot r^{\beta}] =$$

$$= \left(\frac{\Gamma(\beta+1)}{s^{\beta+1}} \right)^N = \frac{\Gamma^N(\beta+1)}{s^{N\beta+N}} \quad \Rightarrow \quad Z_N(L) = \mathcal{L}^{-1}\left[\frac{\Gamma^N(\beta+1)}{s^{N\beta+N}} \right]$$

- aproximace v sedlovém bodě

$$\frac{d}{ds}\left(\frac{\Gamma^N(\beta+1)}{s^{N\beta+N}} e^{sx} \right) = \Gamma^N(\beta+1) \frac{(N\beta+N) s^{N\beta+N-1} e^{sL} - s^{N\beta+N} L e^{sL}}{s^{2N\beta+2N}} \stackrel{!}{=} 0$$

$$s = \frac{N(\beta+1)}{L} \stackrel{\Delta}{=} \lambda \quad \leftarrow \text{sedlový bod } \lambda$$

$$\Rightarrow Z_N(L) \doteq \frac{\Gamma^N(\beta+1)}{\lambda^{N\beta+N}} e^{\lambda L} \quad \& \quad Z_{N-1}(L-r) = \frac{\Gamma^{N-1}(\beta+1)}{\mu^{(N-1)(\beta+1)}} e^{\mu(L-r)}$$

$$\mu = \frac{(N-1)(\beta+1)}{L}$$

Pro $N = L \wedge N \doteq N-1$ (tj. pro velký počet částic):

$$\lambda = \mu = \beta + 1$$

$$\Rightarrow \quad Z_N(L) = \left(\frac{\Gamma(\beta+1)}{(\beta+1)^{\beta+1}} \right)^N e^{\lambda N}$$

$$\Rightarrow g(r) = \Theta(r) \frac{(\beta+1)^{\beta+1}}{\Gamma(\beta+1)} r^{\beta} e^{-(\beta+1)r}$$

Celkem 10 bodů.

Celkem 5 bodů

Konvergence:

$$\mathcal{L}[g(r)] = \frac{(\beta+1)^{\beta+1}}{\Gamma(\beta+1)} \frac{\Gamma(\beta+1)}{(s+\beta+1)^{\beta+1}} = \left(\frac{\beta+1}{s+\beta+1}\right)^{\beta+1} =$$

$$= \left(1 - \frac{s}{s+\beta+1}\right)^{\beta+1} \stackrel{\Delta}{=} G(s) \stackrel{\Delta}{=} G(s|\beta)$$

$$\lim_{\beta\to+\infty} G(s|\beta) = \lim_{\beta\to+\infty}\left(1 - \frac{s}{s+\beta+1}\right)^{\beta+1} = ?$$

$$\lim_{\beta\to\infty} (\beta+1)\cdot \ln\left(1-\frac{s}{s+\beta+1}\right) \stackrel{L'H}{=} -\lim_{\beta\to\infty} \frac{s+\beta+1}{\beta+1}\cdot\frac{s}{(s+\beta+1)^2}(\beta+1)^2 =$$

$$= -s$$

$$\Rightarrow \lim_{\beta\to+\infty} G(s|\beta) = e^{-s}$$

$\mathcal{L}^{-1}[e^{-s}] = \delta(r-1)$ .... částice jsou uspořádány ekvidistantně, tj. jedná se o Diracův částicový systém