2. Fundament. závislosti: $I = I(\rho)$ & $V = V(\rho)$

1934 greenshields (1935 článek)

malý headway!

$$a_k(\tau) = \dot{v}_k(\tau) = \lambda \frac{v_{k-1}(\tilde{\tau}) - v_k(\tilde{\tau})}{\underbrace{[x_{k-1}(\tilde{\tau}) - x_k(\tilde{\tau})]}_{\text{space headway}}^{2}} \leftarrow N, C \qquad \lambda > 0$$

reakční doba

$$\tilde{\tau} = \tau + \widetilde{\Delta\tau}$$

$$\dot{v}_k(\tau) = -\lambda \frac{d}{d\tau} \frac{1}{x_{k-1}(\tilde{\tau}) - x_k(\tilde{\tau})}$$

$$\dot{x}_{k-1}(\tilde{\tau}) = v_k(\tau)$$

$$v_k(\tau) = \frac{-\lambda}{x_{k-1}(\tilde{\tau}) - x_k(\tilde{\tau})} + C_k$$

$$\rho_k^{-1}(\tilde{\tau}) := \overbrace{x_{k-1}(\tilde{\tau}) - x_k(\tilde{\tau})}^{r_k(\tilde{\tau})\ \dots\ \text{prostorový rozestup}} \qquad k = 1, 2, \dots, N \quad \Big\} \Rightarrow \quad \rho_k = \frac{1}{r_k}$$

$\rho_k(\tilde{\tau})$ tzv. individuální hustoty

$$\underbrace{\rho = \frac{1}{N} \sum_{k=1}^{N} \rho_k}_{\text{teoretická alternativa: } \rho = \mathbb{E}\left(\frac{1}{r}\right)}$$

(ρ není klasická hustota)

ρ = aritm. průměr individuálních hustot

!

$$v_k(\tau) = \frac{-\lambda}{x_{k-1}(\tilde{\tau}) - x_k(\tilde{\tau})} + C_k \quad \Big/ \ \frac{1}{N} \sum_k$$

$$v_k(\tau) = -\lambda \rho_k + C_k$$

$$\boxed{V = -\lambda \rho_K + C}$$

Běžná kalibrace pomocí počátečních podmínek:

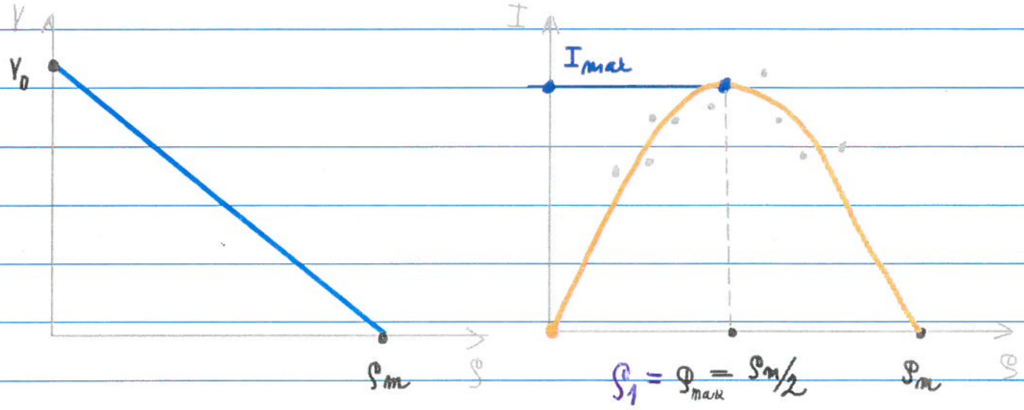
$$\rho = 0 \Rightarrow V = V_0 \quad \text{(opt. hm. rychlost)}$$

$$\underbrace{\rho = \rho_m \Rightarrow V = 0}_{C = V_0 \ \& \ \lambda = \frac{V_0}{\rho_m}}$$

$$I = \rho V$$

$$\boxed{V = V_0\left(1 - \frac{\rho}{\rho_m}\right) \implies I = V_0 \rho \left(1 - \frac{\rho}{\rho_m}\right)}$$

greenshieldsovy vztahy



$$I_{max} = \frac{N}{\tau_N - \tau_1} = \frac{1}{\langle t \rangle} \qquad \langle t \rangle \equiv \mathbb{E}(t) = \int_{\mathbb{R}} t\, f(t)\, dt = \frac{3}{2} \quad [\text{v sekundách}]$$

$$\int_{\mathbb{R}} a^2 x^2 e^{-ax} dx = \frac{2}{a}$$

$$\Rightarrow I_{max} = \frac{2}{3}\, \text{voz}/s = \frac{2}{3}\, \text{vz} \cdot 3600\, h^{-1} = 2400\, \text{voz}/h$$

$$\mathbb{E}\left(\frac{1}{r}\right) = \int_{\mathbb{R}} \frac{1}{r} \cdot g(r)\, dr = \int_0^{+\infty} 40^2 \cdot e^{-40r} dr = 40\, \text{vz}/km$$

$$\rho_1 = \frac{\rho_m}{2} = 40\, \text{vz}/km \implies \rho_m = 80\, \text{vz}/km$$

$$\rho_{max} = 40 \;\&\; I_{max} = 2400 \;\&\; I = \rho V \implies V_{max} = 60\, km/h$$

$$\Rightarrow V_0 = 120\, km/h$$

$$\boxed{V = 120\left(1 - \frac{\rho}{80}\right) \;\&\; I = 120 \rho \left(1 - \frac{\rho}{80}\right)}$$

Klasika: $\int_{\mathbb{R}} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}} \quad / \frac{d}{da}$

$$-\int_{\mathbb{R}} x^2 e^{-ax^2} dx = -\frac{1}{2}\sqrt{\pi}\,\frac{1}{a^{3/2}} \quad \& \quad a = \frac{1}{2s^2}$$

$$\int_{\mathbb{R}} (y-\alpha)^2 e^{-\frac{(y-\alpha)^2}{2s^2}} ds = \int_{\mathbb{R}} z^2 e^{-\frac{z^2}{2s^2}} ds = \sqrt{2\pi}\, s^3 \overset{!}{=} A^{-1}$$

$$\Rightarrow \quad A = \frac{1}{\sqrt{2\pi}\, s^3}$$

$$\rho(x,t) = \frac{\partial N}{\partial x} = \frac{1}{\sqrt{2\pi}\, s^3} \sum_{k=1}^{N} (x-\alpha_k)^2 e^{-\frac{(x-\alpha_k)^2}{2s^2}}$$

$$\lim_{s\to 0_+} \rho(x,t) \overset{\mathcal{D}'}{=} ?$$

$$\left(\lim_{s\to 0_+} \rho(x,t);\, \varphi(x)\right) \overset{\text{def.}}{=} \lim_{s\to 0_+} \left(\rho(x,t);\, \varphi(x)\right) = \left\| \begin{array}{l} \rho(x,t) \text{ je regulární} \\ \text{distribuce a } \varphi(x)\in\mathcal{D} \end{array} \right\| =$$

$$= \lim_{s\to 0_+} \int_{\mathbb{R}} \frac{1}{\sqrt{2\pi}\, s^3} \sum_{k=1}^{N} (x-\alpha_k)^2 e^{-\frac{(x-\alpha_k)^2}{2s^2}} \varphi(x)\, dx =$$

$$= \sum_{k=1}^{N} \lim_{s\to 0_+} \frac{1}{\sqrt{2\pi}\, s^3} \int_{\mathbb{R}} (x-\alpha_k)^2 e^{-\frac{(x-\alpha_k)^2}{2s^2}} \varphi(x)\, dx = \left\| \begin{array}{l} \text{Jakkle ale zaměnit} \\ \text{nelze!} \end{array} \right\| =$$

$$= \left\| \begin{array}{l} y = \frac{x-\alpha_k}{s} \\ dy = \frac{dx}{s} \end{array} \right\| = \sum_{k=1}^{N} \lim_{s\to 0_+} \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} y^2 e^{-\frac{y^2}{2}} \varphi(y\cdot s + \alpha_k)\, dy =$$

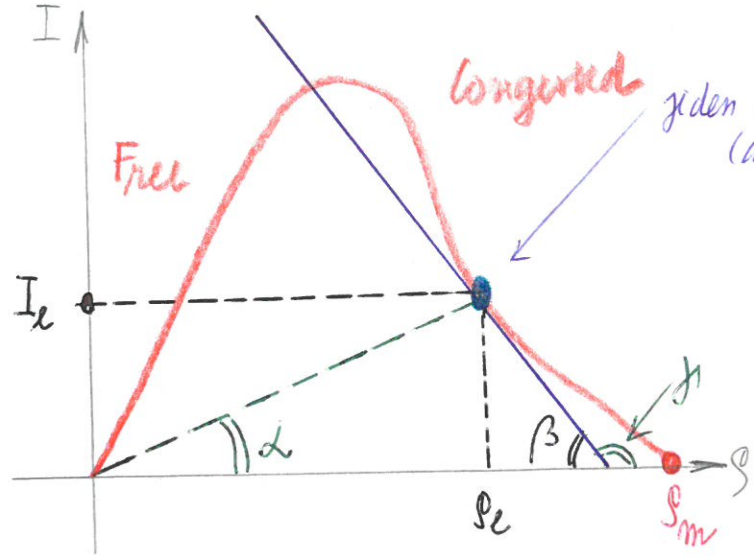
$$= \left\| \, |y^2 e^{-y^2/2} \varphi(y\cdot s+\alpha_k)| \leq K\cdot y^2 \cdot e^{-y^2/2} \in \mathcal{L}(\mathbb{R}) \right\| =$$

z vlastností třídy $\mathcal{D}$

$$= \sum_{k=1}^{N} \varphi(\alpha_k)\cdot \int_{\mathbb{R}} y^2 e^{-y^2/2} dy \cdot \frac{1}{\sqrt{2\pi}} = \sum_{k=1}^{N} \varphi(\alpha_k)\cdot\sqrt{2\pi}\cdot\frac{1}{\sqrt{2\pi}} = \sum_{k=1}^{N} \varphi(\alpha_k) =$$

$$= \left(\sum_{k=1}^{N} \delta_{\alpha_k}(x);\, \varphi(x)\right) \equiv \left(\sum_{k=1}^{N} \delta_{\alpha_k(\tau)}(x);\, \varphi(x)\right) \Rightarrow \lim_{s\to 0_+} \rho(x,y) = \sum_{k=1}^{N} \delta(x-\alpha_k(\tau))$$

Ilustrace geometrických vztahů mezi FD a dopravními veličinami



jeden konkrétní vzorek dopravních dat (aktuálně sledovaný stav dopravy) - např. 100 vozidel

(1.)

$$\mathrm{tg}\,\alpha = \frac{I_e}{\rho_e} = v_e$$, tj. průměrná rychlost sledovaného vzorku

- pokud by individuální rychlosti měly normální rozdělení $N(\mu, \sigma^2)$, pak by $v_e = E(V) = \mu$

(2.) Rychlost kinematické vlny

- víme: $\mu(\rho) = V(\rho) + \rho \cdot \frac{dV}{d\rho}$

$$\frac{dI}{d\rho} = \| \text{tj. tečna ke grafu nahoře v bodě } (\rho_e, I_e) \text{, resp. její tangenta} \| =$$

$$= \frac{d}{d\rho}\left(\rho \cdot V(\rho)\right) = V + \rho \frac{dV}{d\rho} \equiv \mu(\rho)$$

$$\Rightarrow \mu(\rho) = -1 \cdot \mathrm{tg}(\beta) = \mathrm{tg}(\delta)$$

$$\Rightarrow \mathrm{tg}(\delta) = \mu(\rho)$$

Kinematické dopravní vlny

- kinematickou vlnou rozumíme skokovou (rázovou) vlnu v hustotním profilu
- reprezentují pohyb maxima hustoty v prostoročase
- rychlostí kinematické vlny rozumíme rychlost, kterou se porucha (tj. peak hustotního profilu) šíří prostorem

Rámcové odvození

- z $\underbrace{\text{rovnice kontinuity}}_{\frac{\partial \rho}{\partial t}+\frac{\partial I}{\partial x}=0}$, $\underbrace{\text{hydrodynamické alternativy}}_{I=\rho\cdot V}$ a $\underbrace{\text{řešení PDE}}_{}$

rovnice $\frac{\partial f}{\partial x}+\mu\frac{\partial f}{\partial y}=0$

řeš. $\underbrace{f(x,y)=\mathcal{K}(y-\mu x)}_{}$

μ je rychlost šíření funkce f

a fundamentálních hypotéz → $V=V(\rho)$, $I=I(\rho)$

$$\frac{\partial \rho}{\partial t}+\frac{\partial I}{\partial x}=\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}\left(\rho\cdot V(\rho)\right)=\frac{\partial \rho}{\partial t}+V(\rho)\frac{\partial \rho}{\partial x}+\rho\cdot\overbrace{\frac{dV}{d\rho}}^{\triangleq V'}\cdot\frac{\partial \rho}{\partial x}\overset{!}{=}0$$

$$\frac{\partial \rho}{\partial t}+\Big[\underbrace{V(\rho)+\rho\frac{dV}{d\rho}}_{}\Big]\cdot\frac{\partial \rho}{\partial x}=0$$

$\mu(\rho)$ … propagační rychlost kinematické vlny

Zjednodušený vztah pro rychlost $\mu(\rho)$

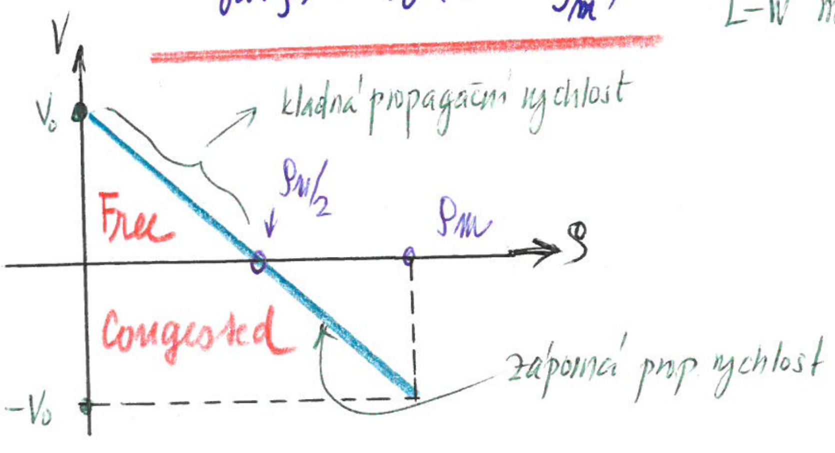
Podmínky:
- •) $V(\rho)$ je lineární funkce (podle Greenschieldse 1934–35)
- •) $\rho=0 \Rightarrow V=V_0$
- •) $\rho=\rho_m \Rightarrow V=0$

$$V=V_0\left(1-\frac{\rho}{\rho_m}\right)$$

$$\Rightarrow V'=\frac{dV}{d\rho}=-\frac{V_0}{\rho_m} \Rightarrow \mu(\rho)=V_0-V_0\frac{\rho}{\rho_m}-\rho\frac{V_0}{\rho_m}$$

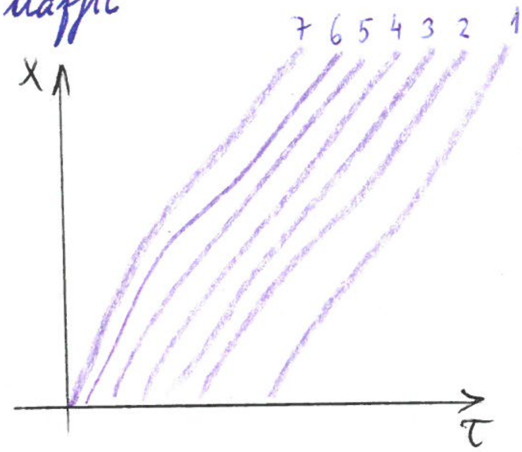
$$\boxed{\mu(\rho)=V_0\left(1-2\frac{\rho}{\rho_m}\right)}$$

Bude se hodit do L-W modelu



Rozbor chování kinematických vln v kontextu třífázové dopravní teorie

I. Free traffic

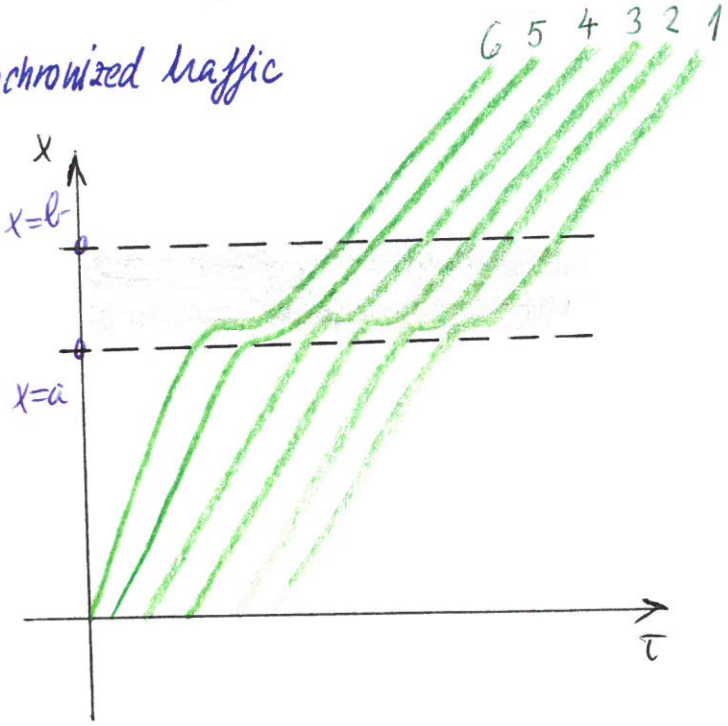


ρ nízké & $\mu(\rho) = V_0\left(1 - 2\frac{\rho}{\rho_m}\right)$

$\Rightarrow \mu \doteq V_0$

- propagační rychlost vlnění splývá s rychlostí vozidel (průměrnou)

II. Synchronized traffic

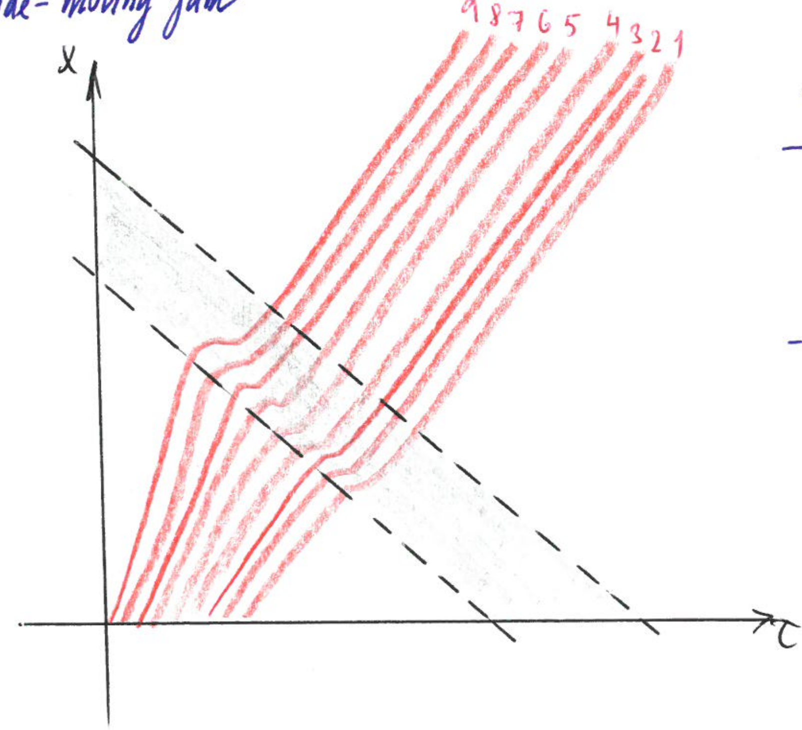


$\rho \doteq \frac{\rho_m}{2} \Rightarrow \mu(\rho) \doteq 0$

nic se nepropaguje

- hustotní peak zůstává na místě, zde v intervalu $\langle a, b \rangle$

III. Wide-moving jam



$\rho \gg \frac{\rho_m}{2} \Rightarrow \mu(\rho) < 0$

- hustotní peaky se sunou v opačném směru než je rychlost vozidel
- uvádí se: běžná rychlost kongesce: $\doteq -15\ \mathrm{km/h}$

$$P(\vec{r}) = \frac{1}{Z_N(L)} e^{-\beta\sum_{k=1}^{N}\varphi(r_k)}\,\Theta(\vec{r})\cdot\delta\Big(L-\sum_{k=1}^{N} r_k\Big)$$

$$g(r_1) = \int_{\mathbb{R}^{N-1}} P(\vec{r})\, d(r_2, r_3, \dots, r_N) = \frac{1}{Z_N(L)}\Theta(r_1) e^{-\beta\varphi(r_1)} \int_{\mathbb{R}^{N-1}} \underbrace{e^{-\beta\sum_{k=2}^{N}\varphi(r_k)}}_{\Theta(r_2, r_3, \dots, r_N)} \delta\Big(L - r_1 - \sum_{k=2}^{N} r_k\Big)\, d(r_2, \dots, r_N) =$$

$$= \Theta(r_1)\,\frac{Z_{N-1}(L-r_1)}{Z_N(L)}\, e^{-\beta\varphi(r_1)}$$

$$\Rightarrow \underline{g(r) = \Theta(r)\,\frac{Z_{N-1}(L-r)}{Z_N(L)}\, e^{-\beta\varphi(r)}}$$

$$\varphi(r) = -\ln r \quad\Rightarrow\quad \underline{g(r) = \Theta(r)\,\frac{Z_{N-1}(L-r)}{Z_N(L)}\, r^{\beta}}$$

$$\mathcal{L}[Z_N(L)] = \int_0^{\infty}\int_{\mathbb{R}^N} \delta\Big(L-\sum_{k=1}^{N} r_k\Big)\cdot\prod_{k=1}^{N}\Theta(r_k)\cdot r_k^{\beta}\, e^{-sL}\, d\vec{r}\, dL = \Big|\, \mathcal{L}[\delta(x-\mu)] = e^{-\mu s} \,\Big| =$$

$$= \int_{\mathbb{R}^N} \prod_{k=1}^{N}\Theta(r_k)\, r_k^{\beta}\, e^{-s\sum_{k=1}^{N} r_k}\, d\vec{r} = \Big(\int_0^{+\infty}\Theta(r)\cdot r^{\beta} e^{-sr}\, dr\Big)^N = \mathcal{L}^N[\Theta(r)\cdot r^{\beta}] =$$

$$= \Big(\frac{\Gamma(\beta+1)}{s^{\beta+1}}\Big)^N = \frac{\Gamma^N(\beta+1)}{s^{N\beta+N}} \quad\Rightarrow\quad \underline{Z_N(L) = \mathcal{L}^{-1}\Big[\frac{\Gamma^N(\beta+1)}{s^{N\beta+N}}\Big]}$$

- aproximace v sedlovém bodě

$$\frac{d}{ds}\Big(\frac{\Gamma^N(\beta+1)}{s^{N\beta+N}} e^{sx}\Big) = \Gamma^N(\beta+1)\,\frac{(N\beta+N)s^{N\beta+N-1} e^{sL} - s^{N\beta+N} L e^{sL}}{s^{2N\beta+2N}} \overset{!}{=} 0$$

$$\underline{s = \frac{N(\beta+1)}{L} \overset{\Delta}{=} \lambda} \quad \leftarrow \text{sedlový bod } \lambda$$

$$\Rightarrow Z_N(L) \doteq \frac{\Gamma^N(\beta+1)}{\lambda^{N\beta+N}} e^{\lambda L} \quad \& \quad Z_{N-1}(L-r) = \frac{\Gamma^{N-1}(\beta+1)}{\mu^{(N-1)(\beta+1)}} e^{\mu(L-r)}$$

$$\mu = \frac{(N-1)(\beta+1)}{L}$$

Pro $N = L \wedge N \doteq N-1$ (tj. pro velký počet částic):

$$\lambda = \mu = \beta+1$$

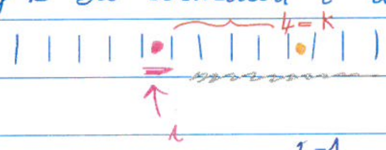
$$\Rightarrow Z_N(L) = \Big(\frac{\Gamma(\beta+1)}{(\beta+1)^{\beta+1}}\Big)^N e^{\lambda N}$$

$$\Rightarrow \underline{g(r) = \Theta(r)\,\frac{(\beta+1)^{\beta+1}}{\Gamma(\beta+1)}\, r^{\beta} e^{-(\beta+1)r}}$$

Celkem 10 bodů.

Distribuce světlostí modelu TASEP

– mezera délky k za obsazenou i-tou buňkou



$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{\langle w|(D+E)^{i-1}\, D\, E^{k-1}\, D\, (D+E)^{N-i-k}|v\rangle}{\langle w|(D+E)^{i-1}\, D\,(D+E)^{N-i}|v\rangle - ⊛} =$$

$$- \underbrace{\langle w|(D+E)^{i-1}\, D\, E^{N-i}|v\rangle}_{⊛}$$

$$= p_i(k) \qquad D = \frac{1}{\beta} \ \& \ E = \frac{1}{\alpha}$$

$$\alpha+\beta = 1 \quad \Rightarrow \quad p_i(k) = \frac{\alpha\beta^{k-1}}{1-\beta^{N-i}}$$

N je velké $\Rightarrow \quad p_i(k) = \alpha\beta^{k-1}$

střední hodnota: $\langle k\rangle = \sum_{l=1}^{\infty} l\, p_i(l) = \alpha \sum_{k=1}^{\infty} k\cdot\beta^{k-1}$

$$\sum_{k=1}^{\infty} k\, x^{k-1} = s(x) \qquad \int s(x)\,dx = \sum_{k=1}^{\infty} x^k = \frac{x}{1-x}$$

$$\Rightarrow s(x) = \left(\frac{x}{1-x}\right)' = \frac{1-x+x}{(1-x)^2} = \frac{1}{(1-x)^2}$$

$$\Rightarrow \langle k\rangle = \cancel{\alpha}\cdot\frac{1}{(1-\beta)^{\cancel{2}}} = \| \alpha+\beta = 1 \| =$$

$$= \frac{1}{\alpha}$$

5

Lighthillův-Whithamův dopravní model (1954)

snaha vysvětlit tvar fundamentálního diagramu ✓ ← I a ϱ zcela nezávislé

snaha vysvětlit tzv. kinematické vlny (tj. časový vývoj hustotních pulzů)

- řešení 1974
- 3 zákl. veličiny: I, ϱ, V $\quad (I = \varrho \cdot V)$

$$I(x,\tau) = \underbrace{I_e(x,\tau)}_{\text{efektivní}} - \underbrace{D \cdot \frac{\partial \varrho(x,\tau)}{\partial x}}_{\text{difúzní porucha}} \qquad (D>0)$$

$$\Rightarrow V(x,\tau) = \frac{I(x,\tau)}{\varrho(x,\tau)} = \underbrace{V_e(x,\tau)}_{V_e(\varrho)} - \frac{D}{\varrho(x,\tau)} \frac{\partial \varrho(x,\tau)}{\partial x}$$

$x \quad \tau$ ⇐ východiska makromodelů

rovnice kontinuity: $\frac{\partial \varrho}{\partial \tau} + \frac{\partial I}{\partial x} = \frac{\partial \varrho}{\partial \tau} + \frac{\partial}{\partial x}\left[\varrho \cdot V_e(\varrho) - D \frac{\partial \varrho}{\partial x}\right] \stackrel{!}{=} 0$

$$\frac{\partial \varrho}{\partial \tau} + \frac{\partial \varrho}{\partial x} V_e(\varrho) + \varrho \frac{dV_e}{d\varrho} \frac{\partial \varrho}{\partial x} - D \frac{\partial^2 \varrho}{\partial x^2} = 0$$

1. $$\boxed{\frac{\partial \varrho}{\partial \tau} + \frac{\partial \varrho}{\partial x}\left[V_e(\varrho) + \varrho \frac{dV_e}{d\varrho}\right] = D \frac{\partial^2 \varrho}{\partial x^2}}$$

Greenshields: **2.** $$\boxed{V_e(\varrho) = V_0 \left(1 - \frac{\varrho}{\varrho_m}\right)}$$

ϱ ●

$$\frac{\partial \varrho}{\partial \tau} + \frac{\partial \varrho}{\partial x} \underbrace{\left[V_0 - V_0 \frac{\varrho}{\varrho_m} - \varrho \frac{V_0}{\varrho_m}\right]}_{V_0\left(1 - 2\frac{\varrho(x,\tau)}{\varrho_m}\right) = \mu(x,\tau)} = D \frac{\partial^2 \varrho}{\partial x^2}$$

← rychlost kinematické vlny

μ ●●

$$\varrho(x,\tau) = \frac{\varrho_m}{2V_0}\left(V_0 - \mu(x,\tau)\right)$$

$$\frac{\partial \varrho}{\partial \tau} = -\frac{\varrho_m}{2V_0} \frac{\partial \mu}{\partial \tau} \quad \& \quad \frac{\partial \varrho}{\partial x} = -\frac{\varrho_m}{2V_0} \frac{\partial \mu}{\partial x} \quad \& \quad \frac{\partial^2 \varrho}{\partial x^2} = -\frac{\varrho_m}{2V_0} \frac{\partial^2 \mu}{\partial x^2}$$

$$\boxed{\frac{\partial \mu}{\partial \tau} + \mu(x,\tau)\,\frac{\partial \mu}{\partial x} = D\,\frac{\partial^2 \mu}{\partial x^2}}$$ konzolidovaný tvar PDE

↓

Burgersova PDE (nelineární)

$$\boxed{\mu(x,\tau) = -\frac{2D}{\varphi(x,\tau)}\,\frac{\partial \varphi}{\partial x}}$$ Cole – Hopf

$$1. \,/\; \frac{\partial \mu}{\partial \tau} = \frac{2D}{\varphi^2(x,\tau)}\,\frac{\partial \varphi}{\partial \tau}\,\frac{\partial \varphi}{\partial x} - \frac{2D}{\varphi(x,\tau)}\,\frac{\partial^2 \varphi}{\partial x\,\partial \tau}$$

$$\mu \,/\; \frac{\partial \mu}{\partial x} = \frac{2D}{\varphi^2(x,\tau)}\left(\frac{\partial \varphi}{\partial x}\right)^2 - \frac{2D}{\varphi(x,\tau)}\,\frac{\partial^2 \varphi}{\partial x^2}$$

$$D \,/\; \frac{\partial^2 \mu}{\partial x^2} = -\frac{4D}{\varphi^3(x,\tau)}\left(\frac{\partial \varphi}{\partial x}\right)^3 + \underbrace{\frac{4D}{\varphi^2(x,\tau)}\,\frac{\partial \varphi}{\partial x}\cdot\frac{\partial^2 \varphi}{\partial x^2} + \frac{2D}{\varphi^2(x,\tau)}\,\frac{\partial \varphi}{\partial x}\,\frac{\partial^2 \varphi}{\partial x^2}}_{6D} - \frac{2D}{\varphi(x,\tau)}\,\frac{\partial^3 \varphi}{\partial x^3}$$

$$\varphi(x,\tau)\,\frac{\partial \varphi}{\partial \tau}\cdot\frac{\partial \varphi}{\partial x} - \varphi^2(x,\tau)\,\frac{\partial^2 \varphi}{\partial x\,\partial \tau} - \cancel{2D\,\frac{\partial \varphi}{\partial x}\left(\frac{\partial \varphi}{\partial x}\right)^2} + 2D\,\frac{\partial \varphi}{\partial x}\,\varphi(x,\tau)\cdot\frac{\partial^2 \varphi}{\partial x^2} =$$

$$= \cancel{-2D\left(\frac{\partial \varphi}{\partial x}\right)^3} + 3D\,\varphi(x,\tau)\,\frac{\partial \varphi}{\partial x}\,\frac{\partial^2 \varphi}{\partial x^2} - D\,\varphi^2(x,\tau)\,\frac{\partial^3 \varphi}{\partial x^3} \quad \Big/ \frac{1}{\varphi(x,\tau)}$$

$$\boxed{\frac{\partial \varphi}{\partial \tau}\,\frac{\partial \varphi}{\partial x} - \varphi(x,\tau)\,\frac{\partial^2 \varphi}{\partial x\,\partial \tau} = D\,\frac{\partial \varphi}{\partial x}\cdot\frac{\partial^2 \varphi}{\partial x^2} - D\,\varphi(x,\tau)\,\frac{\partial^3 \varphi}{\partial x^3}}$$

nelineární III. stupně

$$\frac{\partial \varphi}{\partial x}\left(\frac{\partial \varphi}{\partial \tau} - D\,\frac{\partial^2 \varphi}{\partial x^2}\right) - \varphi(x,\tau)\,\frac{\partial}{\partial x}\left[\frac{\partial \varphi}{\partial \tau} - D\,\frac{\partial^2 \varphi}{\partial x^2}\right] = 0$$

$$\frac{\partial}{\partial x}\left(\frac{\frac{\partial \varphi}{\partial \tau} - D\,\frac{\partial^2 \varphi}{\partial x^2}}{\varphi(x,\tau)}\right) = 0$$

$$\boxed{\frac{\partial \varphi}{\partial \tau} - D\,\frac{\partial^2 \varphi}{\partial x^2} = C\,\varphi(x,\tau)}$$

lin. PDE : lineární alternativa Burgersovy rovnice