1. $P[\eta_L = 0] = 1 - H(L) \Rightarrow H(x) = 1 - P[\eta_x = 0]$

$H(x) = 1 - e^{-2x}(1+2x)$

$h(x) = H'(x) = +2e^{-2x}(1+2x) - 2e^{-2x} = 4xe^{-2x}$

$R_2 = R_1 = R_0 \qquad X_2 = R_0 + R_1 + R_2 \sim (h*h*h)(x)$

$g_2(x) = (h*h*h)(x) \quad / \mathcal{L}$

$G_2(s) = H^3(x) = \left(4\frac{1}{(s+2)^2}\right)^3 = 4^3\frac{1}{(s+2)^6} \quad / \mathcal{L}^{-1}$

$g_2(x) = \frac{4^3}{5!}\Theta(x)\, x^5 e^{-2x}$

$\mathcal{L}[x^n] = \frac{n!}{s^{n+1}}$

$\Downarrow$

$\mathcal{L}[x^n e^{\mu x}] = \frac{n!}{(s+\mu)^{n+1}}$

2. $R_0, R_2, R_4, \dots \sim h(x) \in B$

$R_1, R_3, R_5, \dots \sim f(x) \in B$

$\Rightarrow g_0(x) = h(x) \;\&\; g_1(x) = (h*f)(x) \;\&\; g_2(x) = (h*f*h)(x)$

$\& \dots \& \; g_{2n-1}(x) = (\underbrace{h*f}_{1} * \underbrace{h*f}_{2} \dots * \underbrace{h*f}_{n})(x)$

$g_{2n}(x) = h * (\underbrace{f*h}_{1} * \underbrace{f*h}_{2} * \dots \underbrace{f*h}_{n})(x)$

$\Rightarrow G_{2n-1}(s) = (HF)^n(s) \quad \& \quad G_{2n}(s) = H\cdot(FH)^n(s)$

$R(s) = \mathcal{L}[r(x)] = \mathcal{L}\left(\sum_{k=0}^{\infty} g_k(x)\right) = \sum_{k=0}^{\infty} G_k(s) = \sum_{n=1}^{\infty} G_{2n-1}(s) + \sum_{n=0}^{\infty} G_{2n}(s) =$

$= \sum_{n=1}^{\infty}[H(s)F(s)]^n + H(s)\sum_{n=0}^{\infty}[F(s)H(s)]^n = (1+H(s))\sum_{n=1}^{\infty}[H(s)F(s)]^n + H(s) =$

$= 1 + H(s)\,\frac{H(s)F(s)}{1-H(s)F(s)} + H(s) = \frac{H(s)F(s) + H^2(s)F(s) + H(s) - H^2(s)F(s)}{1-H(s)F(s)} =$

$= H(s)\,\frac{1+F(s)}{1-H(s)F(s)}$

Pokračování př. č. 2

$$f(x) = g(x) \Rightarrow F(s) = H(s) \Rightarrow R(s) = H(s)\,\frac{1+H(s)}{1-H^2(s)} = \frac{H(s)}{1-H(s)}$$

Ověření, že $q \equiv F(s)H(s) < 1$ pro $s > 0$

↳ obě klesající a obě $F(0) = H(0) = 1$ ✓

↑ $F'(s) < 0$ & $H'(s) < 0$

⇑ $F(0) = \mu_0(f)$, $H(0) = \mu_0(g)$

3.

$$h(x) = \theta(x)\int_x^{+\infty} f(y)\,dy = \theta(x)\cdot\left[\int_{\mathbb{R}} f(y)\,dy - \int_{-\infty}^{x} f(y)\,dy\right] =$$

$$= \theta(x)\left[\overbrace{\mu_0(f)}^{\|f\|} - \int_0^x f(y)\,dy\right] \qquad /\!/\ \mathcal{L}$$

$$H(s) = \frac{\|f\|}{s} - \frac{F(s)}{s} ✓$$

$\vec{\mu}$ nechť je momentový kód funkce $f(x)$

$\vec{\lambda}$ ——— " ——— $h(x)$

$$F(s) = \sum_{k=0}^{\infty} (-1)^k \frac{\mu_k}{k!} s^k ✓$$

$$s\cdot H(s) = \|f\| - \sum_{k=0}^{\infty} (-1)^k \frac{\mu_k}{k!} s^k = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\mu_k}{k!} s^k$$

$$H(s) = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{\mu_k}{k!} s^{k-1} = \left\| n = k-1 \right\| =$$

$$= \underbrace{\sum_{n=0}^{\infty} (-1)^n \frac{\mu_{n+1}}{(n+1)!} s^n \overset{!}{=} \sum_{n=0}^{\infty} (-1)^n \lambda_n \cdot \frac{1}{n!} s^n}$$

$$\Downarrow$$

$$\frac{\mu_{n+1}}{(n+1)!} = \frac{\lambda_n}{n!} \Rightarrow \underline{\lambda_n = \frac{\mu_{n+1}}{n+1}} ✓✓$$

4.

$$\mathbb{E}(X) = \int_{\mathbb{R}} x \cdot h(x)\, dx = A \int_0^{+\infty} x^{\alpha+1} e^{-\frac{\beta}{x}} e^{-\lambda x}\, dx = \left\| \begin{matrix} y = \lambda x \\ dy = \lambda\, dy \end{matrix} \right\| =$$

$$= A \int_0^{\infty} \left(\frac{y}{\lambda}\right)^{\alpha+1} e^{-\frac{\beta\lambda}{y}} e^{-y}\, dy \cdot \frac{1}{\lambda} = A \cdot \frac{1}{\lambda^{\alpha+2}} \int_0^{\infty} y^{\alpha+1} e^{-\beta\lambda/y} e^{-y}\, dy =$$

$$= \left| \begin{matrix} a-1 = \alpha+1 & \Rightarrow & a = \alpha+2 \\ x^2/4 = \beta\lambda & \Rightarrow & x^2 = 4\beta\lambda \end{matrix} \right\| = \frac{A}{\lambda^{\alpha+2}} \left(\sqrt{4\beta\lambda}\right)^{\alpha+2} \cdot \frac{1}{2^{\alpha+1}} K_{\alpha+2}\left(2\sqrt{\beta\lambda}\right) =$$

$$= \frac{1}{2}\left(\frac{\lambda}{\beta}\right)^{\frac{\alpha+1}{2}} \frac{K_{\alpha+2}(2\sqrt{\beta\lambda})}{K_{\alpha+1}(2\sqrt{\beta\lambda})} \cdot 2 \cdot (\beta\lambda)^{\frac{\alpha+2}{2}} \frac{1}{\lambda^{\alpha+2}} =$$

$$= \left(\frac{1}{\lambda\beta}\right)^{\frac{\alpha+1}{2}} (\beta\lambda)^{\frac{\alpha+1}{2}} \sqrt{\beta\lambda}\, \frac{1}{\lambda} \frac{K_{\alpha+2}(2\sqrt{\beta\lambda})}{K_{\alpha+1}(2\sqrt{\beta\lambda})} = \sqrt{\frac{\beta}{\lambda}} \frac{K_{\alpha+2}(2\sqrt{\beta\lambda})}{K_{\alpha+1}(2\sqrt{\beta\lambda})} \overset{!}{=} 1$$

$$\mathbb{E}(X^2) = A \int_0^{\infty} x^{\alpha+2} e^{-\beta/x} e^{-\lambda x}\, dx = \dots = \text{analogicky} =$$

$$= \left(\frac{\beta}{\lambda}\right)^{\frac{\alpha+3}{2}} \cdot 2A \cdot K_{\alpha+3}\left(2\sqrt{\beta\lambda}\right) = \dots = \frac{\alpha+\beta+2}{\lambda}$$

aplikace vzorců

$$VAR(X) = \frac{\alpha+\beta+2}{\lambda} - 1$$

5.

Kompresibilita je stoupání (sklon) lineární asymptoty funkce $\Delta(L)$, tj. statistické rigidity.

- namísto asymptotiky $\Delta(L)$ v nekonečnu se pracuje s Taylorovým rozvojem Laplaceova obrazu $\Delta(L) = \mathcal{L}[\Delta(L)]$

$$\Delta(L) \doteq \chi \cdot L + \varkappa \quad \text{pro } L \to +\infty \;\; // \; \mathcal{L}$$

$$\Delta(s) \doteq \frac{\chi}{s^2} + \frac{\varkappa}{s} \quad \text{na } U_\delta(0)$$

χ... kompresibilita