$$g_n(x) = \frac{e^{-2nx}}{\cosh^2(nx)+\sinh^2(nx)} = \frac{e^{-2nx}}{\left(\frac{e^{nx}+e^{-nx}}{2}\right)^2+\left(\frac{e^{nx}-e^{-nx}}{2}\right)^2} =$$

$$= \frac{4\cdot e^{-2nx}}{e^{2nx}+e^{-2nx}+2+e^{2nx}+e^{-2nx}-2} = \frac{1}{2}\frac{4e^{-2nx}}{e^{2nx}+e^{-2nx}} = \frac{2}{e^{4nx}+1}$$

ta má maximum v nule! (stále klesá)

$$\Rightarrow \forall x \in \mathbb{R}_0^+ : \left| \frac{(3n-4)!!!}{3^n \cdot n!} g_n(x) \right| \leq \frac{(3n-4)!!!}{3^n\cdot n!}$$

pokus o aplikaci Weierstrasova kritéria

název kritéria

Raabe:

$$\lim_{n\to\infty} n\left(1-\frac{(3n-1)!!!}{3^{n+1}(n+1)!}\cdot\frac{3^n\cdot n!}{(3n-4)!!!}\right) =$$

$$= \lim_{n\to\infty} n\left(1-\frac{3n-1}{3n+3}\right) = \lim_{n\to\infty} n\,\frac{3n+3-3n+1}{3n+3} =$$

$$= \lim_{n\to\infty}\frac{4n}{3n+3} = \frac{4}{3} > 1 \quad (\text{hurá } \text{:-)})$$

$\Rightarrow \sum_n \frac{(3n-4)!!!}{3^n\cdot n!}$ poštovky konvergují $\Rightarrow$ podle Weierstrasse konvergují $\sum_n \frac{(3n-4)!!!}{3^n\cdot n!} g_n(x)$ stejnoměrně na $\mathbb{R}_0^+$

$$y'' - \operatorname{cotg}(x)\, y' = \frac{3}{\sin^3(x)}$$

9 bodů

$$z = y'$$

$$z' - \operatorname{cotg}(x) \cdot z = \frac{3}{\sin^3(x)} \qquad \Big/ \cdot \frac{1}{\sin x}$$

$$\left(z \cdot \frac{1}{\sin x} \right)' = \frac{3}{\sin^4(x)}$$

$$\int \frac{3}{\sin^4(x)}\, dx = \left| \begin{matrix} t = \operatorname{tg}(x) \\ dx = \frac{dt}{1+t^2} \end{matrix} \quad \sin^2 x = \frac{t^2}{1+t^2} \right| = \int \frac{3(1+t^2)^2}{t^4} \cdot \frac{1}{1+t^2}\, dt =$$

$$= 3 \int \frac{1+t^2}{t^4}\, dt = -t^{-3} - 3t^{-1} + C = -\frac{\cos^3 x}{\sin^3 x} - 3\frac{\cos x}{\sin x} + C$$

$$z \cdot \frac{1}{\sin x} = C - \frac{\cos^3 x}{\sin^3 x} - 3\frac{\cos x}{\sin x}$$

$$z(x) = C\sin(x) - 3\cos x - \frac{\cos^3 x}{\sin^2 x}$$

$$y(x) = C\cos(x) - 3\sin(x) - \int \frac{(1-\sin^2 x)\cos x}{\sin^2(x)}\, dx = \left| \begin{matrix} t = \sin x \\ dt = \cos x\, dx \end{matrix} \right| =$$

$$= C\cos(x) - 3\sin(x) - \int \frac{1-t^2}{t^2}\, dt = C\cos x - 3\sin x + \frac{1}{\sin x} + \sin x + D$$

$$y(x) = C\cos(x) + D - 2\sin(x) + \frac{1}{\sin(x)} \qquad I = (0; \pi)$$

7b

$$g_n(x) = e^n \underbrace{(2x-3x^2)^n}_{\overset{\Delta}{=} f_n(x)} \qquad Dom(g_n) = \langle 0; 2/3 \rangle$$

$$f_n'(x) = n\,(2x-3x^2)^{n-1} \cdot (2-6x) \overset{!}{=} 0 \Rightarrow 3 \text{ stac. body} \begin{cases} x_0 = 0 \\ x_1 = 2/3 \\ x_2 = 1/3 \end{cases}$$

$$f_n(0) = 0 \quad \& \quad f_n(2/3) = \left(2 \cdot \frac{2}{3} - 3 \cdot \frac{4}{9}\right)^n = 0$$

$$\& \quad f_n(1/3) = \left(\frac{2}{3} - 3 \cdot \frac{1}{9}\right)^n = \left(\frac{1}{3}\right)^n \quad \Delta \quad f_n(x) \in C^1(\mathbb{R})$$

$\Rightarrow$ bodem maxima je $x_2 = 1/3$

$$\Rightarrow \forall x \in \langle 0; 2/3 \rangle : \quad |f_n(x)| \leqq f_n(1/3) = \left(\frac{1}{3}\right)^n$$

$$|g_n(x)| \leqq e^n \cdot \left(\frac{1}{3}\right)^n$$

$$\Rightarrow \sup_{x \in \langle 0; 2/3 \rangle} |g_n(x) - g(x)| = e^n \cdot \left(\frac{1}{3}\right)^n \overset{\Delta}{=} \sigma_n$$

$$(*) \quad g(x) = \lim_{n\to\infty} e^n (2x-3x^2)^n = 0 \Leftarrow 0 \leqq e^n (2x-3x^2)^n \leqq e^n \cdot \left(\frac{1}{3}\right)^n$$

výpočet limitní funkce

$$\& \lim_{n\to\infty} e^n \left(\frac{1}{3}\right)^n = 0 \Leftarrow \underbrace{\frac{e}{3} < 1}_{\text{ale jen těsně}}$$

:)

$$\lim_{n\to\infty} \sigma_n = 0 \Rightarrow g_n(x) \overset{\langle 0; 2/3 \rangle}{\rightrightarrows} 0$$

10b

$\hat{L}(y(x)) = y' - 2y = 0 \quad \Rightarrow \quad \Omega_0 = [e^{2x}]_\lambda$

Ze zadání plyne, že $e^{2x} \in \tilde{\Omega}_0 \qquad \tilde{\Omega}_0 = \{y(x) \in C^3(I) : \hat{K}(y(x)) = 0\}$

Snížení řádu I.

$y = k \cdot e^{2x} \quad y' = k'e^{2x} + 2k \cdot e^{2x} \quad y'' = k''e^{2x} + 4k'e^{2x} + 4ke^{2x}$

$y''' = k'''e^{2x} + 2k''e^{2x} + 4k''e^{2x} + 8k'e^{2x} + 4k'e^{2x} + 8ke^{2x}$

Dosazení:

$x^2(k''' + 6k'' + 12k' + 8k) + (2x - 6x^2)(k'' + 4k' + 4k) +$
$+ (12x^2 - 8x - 2)(k' + 2k) + (4 + 8x - 8x^2)k = 0$

$x^2 k''' + (6x^2 + 2x - 6x^2)k'' + (12x^2 + 8x - 24x^2 + 12x^2 - 8x - 2)k' + 0 \cdot k = 0$

$x^2 k''' + 2xk'' - 2k' = 0 \qquad w = k'$

$x^2 w'' + 2xw' - 2w = 0$

Snížení řádu II.

- tuhle rovnici zjevně řeší funkce $g(x) = x$
- proto: $\quad w(x) = x \cdot a(x)$

$w' = a'x + a \quad \& \quad w'' = a''x + 2a'$

Dosazení:

$a''x^3 + 2a'x^2 + 2x^2a' + 2xa - 2xa = 0$

$a''x^3 + 4x^2a' = 0$

$a'' + \frac{4}{x}a' = 0 \quad | \text{ I.F. } x^4$

$(a' \cdot x^4)' = \hat{C}$

$a' = \frac{C}{x^4} \Rightarrow a(x) = \frac{D}{x^3} + E \Rightarrow w(x) = \frac{D}{x^2} + Ex$

$\Rightarrow k(x) = \frac{G}{x} + Hx^2 + J \quad \Rightarrow \qquad G, H, J \in \mathbb{R}$

stačí jedno z toho

$y(x) = \frac{G}{x}e^{2x} + Hx^2e^{2x} + Je^{2x} \; ; \quad \mathrm{Dom}(y) = \mathbb{R}^+$

$y(x) \in \left[\frac{e^{2x}}{x}; x^2e^{2x}; e^{2x}\right]_\lambda \equiv \tilde{\Omega}_0$

$\dim(\tilde{\Omega}_0) = 3 \quad \text{vs.} \quad \dim(\Omega_0) = 1$

$$R^{-1} = \lim_{n\to\infty} \frac{(4n+6)!!!!}{(n+2)!} \cdot \frac{(n+1)!}{(4n+2)!!!!} = \lim_{n\to\infty} \frac{4n+6}{n+2} = 4 \Rightarrow R = 1/4$$

to by platilo pro řadu $\sum a_n x^n$

my: $|x-1|^{2n} = \frac{1}{4}$ (určujeme hraniční body oboru konvergence)

$|x-1| = \frac{1}{2}$

$x = \{\frac{1}{2}; \frac{3}{2}\} \Rightarrow \bar{O} = \langle \frac{1}{2}; \frac{3}{2} \rangle$

správné krajní body ! ale $O = ?$

Krajní řady:

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(4n+2)!!!!}{(n+1)!} \left(\frac{1}{4}\right)^n$$

← správný tvar krajních řad

Raabe:

$$\lim_{n\to\infty} n\left(1 - \frac{(4n+6)!!!!}{(n+2)!} \cdot \frac{4^n}{4^{n+1}} \cdot \frac{(n+1)!}{(4n+2)!!!!}\right) =$$

$$= \lim_{n\to\infty} n\left(1 - \frac{4n+6}{4n+8}\right) = \lim_{n\to\infty} n\left(\frac{4n+8-4n-6}{4n+8}\right) = \frac{1}{2} \in (0,1)$$

$$\Rightarrow O = \langle \frac{1}{2}; \frac{3}{2} \rangle$$