Stačí vyšetřit extrém funkce $f(x) = x^2 e^{3-3x^2}$

$$f'(x) = (2x - x^2 \cdot 6x)\, e^{3-3x^2} \overset{!}{=} 0$$

nebo

$t = x^2$

$f(t) = t e^{3(1-t)}$

$f'(t) = e^{3(1-t)}[1-3t]$

$t = 1/3$

$\Rightarrow x = 1/\sqrt{3}$

$$1 - 3x^2 = 0$$

$$x = 1/\sqrt{3} \;\ldots\; \text{zjevně jde o maximum}$$

$\Leftarrow f(0) = 0$ & $\lim_{x\to\infty} f(x) = 0$

& $f(x) \in C(\mathbb{R}^+)$

$$\left| (-1)^{n+1} \frac{(3n+1)!!!}{(n+1)!} \left(x^2 e^{3-3x^2}\right)^n \right| \leqq \frac{(3n+1)!!!}{(n+1)!} \left(f(1/\sqrt{3})\right)^n =$$

$$= \frac{(3n+1)!!!}{(n+1)!} \left(\frac{e^{2}}{3}\right)^n \quad !$$

Raabe:

$$\lim_{n\to\infty} n \left(1 - \frac{(3n+4)!!!}{(n+2)!} \cdot \frac{3^n}{3^{n+1}} \cdot \frac{(n+1)!}{(3n+1)!!!}\right) =$$

$$= \lim_{n\to\infty} n \left(1 - \frac{3n+4}{3n+6}\right) = \lim_{n\to\infty} n \left(\frac{2}{3n+6}\right) = \frac{2}{3} \in (0,1)$$

$$\Rightarrow \sum_n \frac{(3n+1)!!!}{(n+1)!} \frac{1}{3^n} \text{ nekonverguje}$$

$\Rightarrow$ W- kritérium nelze užít ani v jednom případě

stačí podílové

$$\lim_{n\to+\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to+\infty} \left(\frac{3n+4}{n+2} \frac{e^2}{3}\right) = e^2 > 1 \Rightarrow D$$

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Pomocný výpočet č. 1

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$$\int \frac{1}{x(x-2)}\,dx = \int \left(-\frac{1/2}{x} + \frac{1/2}{x-2} \right) dx = -\frac{1}{2}\ln(x) + \frac{1}{2}\ln|2-x|$$

– hledáme řešení na okolí bodu $x = 1 \Rightarrow |x-2| = 2-x$

$$y' + \frac{y}{x(x-2)}\,y = \frac{3x}{\sqrt{2-x}} \quad // \quad I.F. = \sqrt{\frac{2-x}{x}}$$

$$\left(y \cdot \sqrt{\frac{2-x}{x}} \right)' = 3\sqrt{x}$$

$$\left(y \cdot \sqrt{\frac{2-x}{x}} \right)' = \left(2x^{3/2} + c \right)'$$

$$y(x) = C \cdot \sqrt{\frac{x}{2-x}} + \frac{2x^2}{\sqrt{2-x}}$$

$$y(1) = C + 2 \overset{!}{=} 7 \quad \Rightarrow \quad C = 5$$

$$y(x) = \frac{5\sqrt{x} + 2x^2}{\sqrt{2-x}}$$

$$\mathrm{Dom}(y) = (0,2)$$

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$$R^{-1} = \lim_{n\to\infty} \frac{(2n+1)!}{[(n+1)!]^2} \cdot \frac{[n!]^2}{(2n-1)!} = \lim_{n\to\infty} \frac{(2n+1)\cdot 2n}{(n+1)(n+1)} = 4 \Rightarrow R = 1/4$$

– krajní body obom konvergence : $\frac{3}{4}$ & $\frac{5}{4}$

spravně určené krajní body

$$x_L = \frac{3}{4} \Rightarrow \sum_{n=1}^{\infty} \frac{(2n-1)!}{(n!)^2}\left(-\frac{1}{4}\right)^n$$

$$x_p = \frac{5}{4} \Rightarrow \sum_{n=1}^{\infty} \frac{(2n-1)!}{(n!)^2}\left(\frac{1}{4}\right)^n$$

TVAR KRAJNÍCH ŘAD

Raabe:

$$\lim_{n\to\infty} n\left(1 - \left|\frac{a_{n+1}}{a_n}\right|\right) = \lim_{n\to\infty} n\left(1 - \frac{(2n+1)!}{(n+1)!(n+1)!} \cdot \frac{n!\cdot n!}{(2n-1)!} \cdot \frac{4^n}{4^{n+1}}\right) =$$

$$= \lim_{n\to\infty} n\left(1 - \frac{(2n+1)\cdot 2n}{4(n+1)^2}\right) = \lim_{n\to\infty} n\left(1 - \frac{4n^2+2n}{4n^2+8n+4}\right) =$$

$$= \lim_{n\to\infty} n\left(\frac{6n+4}{4n^2+8n+4}\right) = \frac{6}{4} = \frac{3}{2} > 1$$

$$\Rightarrow \quad G = \left\langle \frac{3}{4}; \frac{5}{4} \right\rangle$$

Dirichletovské členění:

$$f_n(x) = (-1)^n \quad \& \quad g_n(x) = \frac{x}{x^2+n^2}$$

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1) $\sum(-1)^n$ má omezené částečné součty

2) $g_{n+1}(x) < g_n(x) \ \dots \quad \forall x \in \langle 0; +\infty)$

$$g_{n+1}(x) = \frac{x}{x^2+(n+1)^2} < \frac{x}{x^2+n^2} = g_n(x)$$

$$x^2+n^2 < x^2+n^2+2n+1$$

$$0 < 2n+1$$

3) $g_n(x) \overset{?}{\rightrightarrows} 0$

$$\lim_{n\to\infty} \frac{x}{x^2+n^2} = 0$$ ← explicitně uvedeno

$$\sigma_n = \sup_{x\in\mathbb{R}} \frac{x}{x^2+n^2}$$

$$g_n'(x) = \frac{x^2+n^2-2x^2}{(x^2+n^2)^2} = \frac{n^2-x^2}{(x^2+n^2)} \overset{!}{=} 0$$

$$x = \cancel{\pm} n$$

$g_n(0) = 0 \ \wedge \ g_n(x) \in C(\mathbb{R}^+) \ \wedge \ \lim_{x\to+\infty} g_n(x) = 0$

$\Rightarrow$ $x = n$ je globální maximum

$$\sigma_n = g_n(n) = \frac{n}{n^2+n^2} = \frac{1}{2n}$$

$\lim_{n\to\infty} \sigma_n = \lim_{n\to\infty} \frac{1}{2n} = 0 \Rightarrow$ ze supremálního kritéria $g_n(x) \overset{\mathbb{R}^+}{\rightrightarrows} 0$

Všechny 3 předpoklady Dirichletova kritéria splněny $\Rightarrow$

$\Rightarrow$ řada konverguje na $\langle 0; +\infty)$ stejnoměrně

$$\lim_{n\to\infty}\left(x+\frac{x^2n^2}{x^2n^2+xn+1}\right)=\underline{x+1}$$ limitní funkce

$$h_n(x)=g_n(x)-(x+1)=x+\frac{x^2n^2}{x^2n^2+xn+1}-x-1=\frac{-xn-1}{x^2n^2+xn+1}$$

$$\sigma_n=\sup_{x>0}|h_n(x)|=\sup_{x>0}\frac{xn+1}{x^2n^2+xn+1}$$

$$\left(\frac{xn+1}{x^2n^2+xn+1}\right)'=\frac{n(x^2n^2+xn+1)-(xn+1)(2xn^2+n)}{(x^2n^2+xn+1)^2}\overset{!}{=}0$$

$$x^2n^2+xn+1-(xn+1)(2xn^2+n)=0$$

$$x^2n^2+xn+1-2x^2n^2-xn-2xn-1=0$$

$$xn(xn+2)=0$$

$$\underline{x=0 \qquad x=-2/n} \notin (0,+\infty)\ !$$

$\Rightarrow\ \forall x\in\mathbb{R}^+:\ h_n'(x)<0 \Rightarrow$ funkce $h_n(x)$ pro $x\to+\infty$ klesá k nule

$\Rightarrow\ \sigma_n=|h_n(0)|=1$

$\lim_{n\to\infty}\sigma_n=1\neq 0 \Rightarrow$ posloupnost na $(0,+\infty)$ stejnoměrně nekonverguje