5

$$\alpha = 135^\circ \Rightarrow tg(\alpha) = -1 \Rightarrow y'(x_0) \overset{!}{=} -1$$

$$x_0 = ?$$

$$y' = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = -\frac{2+2x+4y}{-2+4x+10y} \overset{!}{=} -1$$

$$2 + 2x + 4y = -2 + 4x + 10y$$

$$x = -3y + 2$$

$$8 + 2(-3y+2) + (-3y+2)^2 - 2y + 4y(-3y+2) + 5y^2 = 0$$

$$8 - 6y + 4 + 9y^2 - 12y + 4 - 2y - 12y^2 + 8y + 5y^2 = 0$$

$$2y^2 - 12y + 16 = 0$$

$$y^2 - 6y + 8 = 0$$

$$(y-2)(y-4) = 0$$

Dvě řešení:

$$\underline{(-4, 2) \ \& \ (-10, y=4)}$$

$$x = a\rho\cos\varphi$$
$$y = b\rho\sin\varphi$$

$$\det\left(\frac{D(x,y)}{D(\rho,\varphi)}\right) = \begin{vmatrix} a\cos\varphi & -a\rho\cos\varphi \\ b\sin\varphi & b\rho\cos\varphi \end{vmatrix} = ab\rho \overset{!}{\neq} 0$$

$$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^4 \leq \frac{2xy}{ab}$$

$$\rho^8 \leq 2\rho^2\cos\varphi\sin\varphi$$

$$\rho^6 \leq \sin(2\varphi) \Rightarrow \sin(2\varphi) \overset{!}{>} 0 \Rightarrow \begin{cases} \varphi \in (0; \frac{\pi}{2}) \\ \varphi \in (\pi; \frac{3\pi}{2}) \end{cases}$$

$$\int_A x^2y^2\, d(x,y) = 2\int_0^{\pi/2}\int_0^{\sqrt[6]{\sin(2\varphi)}} a^2b^2\rho^4\cos^2\varphi\sin^2(\varphi)\cdot ab\rho\, d\rho\, d\varphi =$$

chybné meze ⇒ konec

chybný jakobián ⇒ konec

$$= \frac{2}{6}a^3b^3\int_0^{\pi/2}\cos^2(\varphi)\sin^2(\varphi)\cdot\left[\rho^6\right]_0^{\sqrt[6]{\sin(2\varphi)}} d\varphi =$$

$$= \frac{1}{3}a^3b^3\int_0^{\pi/2}\frac{1}{4}\sin^2(2\varphi)\cdot\sin(2\varphi)\, d\varphi = \left\| \begin{matrix} u = \cos(2\varphi) \\ du = -2\sin(2\varphi)d\varphi \end{matrix} \right\| =$$

$$= \frac{1}{24}a^3b^3\int_{-1}^{1}(1-u^2)\, du = \frac{1}{12}a^3b^3\int_0^1(1-u^2)\, du =$$

$$= \frac{1}{12}a^3b^3\left[u - \frac{u^3}{3}\right] = \frac{1}{18}a^3b^3$$

$$L(x,y,z) = 4 + x^2z^2y + \lambda(16x^4 + 16z^4 + y^2)$$

$$\left.\begin{aligned} \frac{\partial L}{\partial x} &= 2xyz^2 + 64\lambda x^3 \overset{!}{=} 0 \\ \frac{\partial L}{\partial y} &= x^2z^2 + 2\lambda y \overset{!}{=} 0 \\ \frac{\partial L}{\partial z} &= 2x^2yz + 64\lambda z^3 \overset{!}{=} 0 \end{aligned}\right\} \Rightarrow \quad \begin{aligned} &x,y,z>0 \Rightarrow \lambda<0 \\ &\text{symetrie: } x = z \end{aligned}$$

je-li stacion. bod určen špatně, dále se neopravuje!

Jediný stacionární bod: $\vec{a} = (1,4,1)$ & $\lambda = -\frac{1}{8}$

$$\frac{\partial^2 L}{\partial x^2} = 2yz^2 + 3\cdot 64\lambda x^2 \qquad \frac{\partial^2 L}{\partial y^2} = 2\lambda \qquad \frac{\partial^2 L}{\partial z^2} = 2x^2y + 3\cdot 64\lambda z^2$$

$$\frac{\partial^2 L}{\partial x\partial y} = 2xz^2 \qquad \frac{\partial^2 L}{\partial x\partial z} = 4xyz \qquad \frac{\partial^2 L}{\partial y\partial z} = 2x^2z$$

$$\mathbb{L} = \begin{pmatrix} -16 & 2 & 16 \\ 2 & -1/4 & 2 \\ 16 & 2 & -16 \end{pmatrix}$$

Redukce druhého totálního diferenciálu: $g(x,y,z) = 16x^4 + 16z^4 + y^2 - 48$

$$\operatorname{grad} g = (64x^3; 2y; 64z^3) \Rightarrow \operatorname{grad} g(\vec{a}) = (64, 8, 64)$$

$$dg_{(1,4,1)} = 64dx + 8dy + 64dz = 0 \Rightarrow dy = -8(dx+dz)$$

$$d^2L^{(red)}_{\vec{a}}(dx,dz) = -64(dx^2 + dz^2 + dx\,dy^{z}) =$$

$$= 64(dx; dz)\begin{pmatrix} -1 & -1/2 \\ -1/2 & -1 \end{pmatrix}\begin{pmatrix} dx \\ dy \end{pmatrix}$$

$$\Delta_1 = -1 < 0 \quad \& \quad \Delta_2 = \begin{vmatrix} -1 & -1/2 \\ -1/2 & -1 \end{vmatrix} = 1 - \frac{1}{4} = \frac{3}{4} > 0$$

podle Sylvesterova kritéria: $d^2L^{(red)}_{\vec{a}}(dx,dz) < 0$

$\Rightarrow$ $\vec{a} = (1,4,1)$ je O.L.V. maximum

musí být odůvodněno korektně

5b

$\tilde{H}(u, v)$

$x\,y\,z \quad x\,y\,z$

$dH_{\vec{b}}(h_u, h_v) = 3h_u - 2h_v \Rightarrow \frac{\partial H}{\partial u}(\vec{b}) = 3 \;\&\; \frac{\partial H}{\partial v}(\vec{b}) = -2$

$du_{\vec{a}}(h_x, h_y, h_z) = h_x - 3h_y + h_z \Rightarrow \frac{\partial u}{\partial x}(\vec{a}) = 1 \;\&\; \frac{\partial u}{\partial y}(\vec{a}) = -3 \;\&\; \frac{\partial u}{\partial z}(\vec{a}) = 1$

$dv_{\vec{a}}(h_x, h_y, h_z) = -h_y + 5h_z \Rightarrow \frac{\partial v}{\partial x}(\vec{a}) = 0 \;\&\; \frac{\partial v}{\partial y}(\vec{a}) = -1 \;\&\; \frac{\partial v}{\partial z}(\vec{a}) = 5$

2 věty o derivaci složené funkce:

$$\frac{\partial \tilde{H}}{\partial x}(\vec{a}) = \frac{\partial H}{\partial u}(\vec{b}) \cdot \frac{\partial u}{\partial x}(\vec{a}) + \frac{\partial H}{\partial v}(\vec{b}) \cdot \frac{\partial v}{\partial x}(\vec{a}) = 3 \cdot 1 - 2 \cdot 0 = 3$$

$$\frac{\partial \tilde{H}}{\partial y}(\vec{a}) = \frac{\partial H}{\partial u}(\vec{b}) \cdot \frac{\partial u}{\partial y}(\vec{a}) + \frac{\partial H}{\partial v}(\vec{b}) \cdot \frac{\partial v}{\partial y}(\vec{a}) = 3(-3) + (-2)(-1) = -7$$

$$\frac{\partial H}{\partial z}(\vec{a}) = \frac{\partial H}{\partial u}(\vec{b}) \cdot \frac{\partial u}{\partial z}(\vec{a}) + \frac{\partial H}{\partial v}(\vec{b}) \cdot \frac{\partial v}{\partial z}(\vec{a}) = 3 \cdot 1 - 2 \cdot 5 = -7$$

$$\operatorname{grad} \tilde{H}(\vec{a}) = (3; -7; -7)$$

$$\|\vec{s}\|^2 = 1 + 4 + 4 \Rightarrow \|\vec{s}\| = 3$$

$$\frac{\partial \tilde{H}}{\partial \vec{s}}(\vec{a}) = \frac{1}{3} \langle \operatorname{grad} \tilde{H}(\vec{a}) \,|\, \vec{s} \rangle = \frac{1}{3}(3 \cdot 1 - 7 \cdot 2 + 7 \cdot 2) = 1$$

$$\frac{\partial H}{\partial \vec{s}}(\vec{a}) = 1$$

~~10,8~~ 9

$$x^2 \frac{\partial^2 u}{\partial x^2} - y^2 \frac{\partial^2 u}{\partial y^2} + x(1+8x^2y^2)\frac{\partial u}{\partial x} - y(1+8x^2y^2)\frac{\partial u}{\partial y} = 0$$

$$x^2 \frac{\partial^2 u}{\partial x^2} - y^2 \frac{\partial^2 u}{\partial y^2} + (1+8x^2y^2)\left(x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y}\right) = 0$$

$$\xi = xy \qquad\qquad \Delta = \begin{vmatrix} y & x \\ 1/y & -\frac{x}{y^2} \end{vmatrix} = -2\frac{x}{y} \neq 0$$
$$\eta = x/y$$

dohromady jeden bod

$$H_{reg} = \{(x,y)\in E^2 : x \neq 0 \land y \neq 0\}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \xi} y + \frac{\partial u}{\partial \eta}\frac{1}{y} \qquad\qquad \frac{\partial u}{\partial y} = \frac{\partial u}{\partial \xi} x - \frac{\partial u}{\partial \eta}\frac{x}{y^2}$$

$$\frac{\partial^2 u}{\partial x^2} = y^2 \frac{\partial^2 u}{\partial \xi^2} + \frac{1}{y^2}\frac{\partial^2 u}{\partial \eta^2} + 2\frac{\partial^2 u}{\partial x \partial y}$$

$\frac{\partial^2 u}{\partial \xi \partial \eta}$

$$\frac{\partial^2 u}{\partial y^2} = x^2 \frac{\partial^2 u}{\partial \xi^2} + \frac{x^2}{y^4}\frac{\partial^2 u}{\partial \eta^2} - 2\frac{x^2}{y^2}\frac{\partial^2 u}{\partial \xi \partial \eta} + \frac{2x}{y^3}\frac{\partial u}{\partial \eta}$$

Dosazení:

$$\frac{\partial^2 u}{\partial \xi^2}(x^2y^2 - x^2y^2) + \frac{\partial^2 u}{\partial \eta^2}\left(\frac{x^2}{y^2} - \frac{x^2}{y^2}\right) + \frac{\partial^2 u}{\partial \xi \partial \eta}(2x^2 + 2x^2) +$$
$$+ \frac{\partial u}{\partial \xi}(xy + 8x^3y^3 - xy - 8x^3y^3) + \frac{\partial u}{\partial \eta}\left(-\frac{2x}{y} + \frac{x}{y} + 8x^3y + \frac{x}{y} + 8x^3y\right) = 0$$

$$4x^2 \frac{\partial^2 u}{\partial \xi \partial \eta} + 16x^3y\frac{\partial u}{\partial \eta} = 0$$

$$\frac{\partial^2 u}{\partial \xi \partial \eta} + 4\xi \frac{\partial u}{\partial \eta} = 0$$

$$v := \frac{\partial u}{\partial \eta} \qquad \frac{\partial v}{\partial \xi} + 4\xi v = 0 \quad \Rightarrow \quad \frac{\partial v}{\partial \xi} = -4\xi v$$

$$\frac{1}{v}\partial v = -4\xi\, d\xi$$ → zápis OK? ∂v a dξ

$$\ln|v| = -2\xi^2 + c$$
$$v(\xi,\eta) = c(\eta)\cdot e^{-2\xi^2}$$

$$u(\xi,\eta) = D(\eta)e^{-2\xi^2} + E(\xi)$$

dohromady 1 bod

$$u(x,y) = D\left(\frac{x}{y}\right)e^{-2x^2y^2} + E(xy)$$ [redacted]