socio:physical modelling

mathematical modelling of socio-physical dynamics

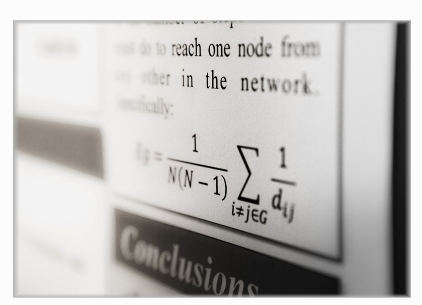
mmsd

IN A CROWD

Movements of an arbitrary group of humans (pedestrians in corridors, drivers on freeways, people travelling on airport conveyors, drivers maneuvering when parking, etc.) show many common features originated from a group dynamics. For purposes of socio-physical science the *Agent Group* is understood as a self-organized system whose individual agents are influenced by other agents in the group. It means that each agent adapts its behavior to the behavior of the rest of his/her group. Such an influence is naturally restricted to the interactions with agents occurring in the close neighborhood (middle-ranged interactions). Moreover, the decision-making process of a moving agent is influenced by the various factors (individuality of the agent, actual mental strain, control signals, information inflow, random factors, and so on). Typically, the mediated collective decision-making of a group leads to *effects of crowding*, i.e. to the formation of congested states when the movement of one agent is strongly restricted by other agents.

It is obvious that mutual interactions among the agents cause the changes in the system dynamics, which finally results in associated changes of macroscopic quantities for the system investigated. Furthermore, macroscopic relations describing the global behavior of transport systems influence significantly the microscopic structure of the system. Such a structure is, as understandable, of statistical nature, which is caused by the individuality of each agent. Whereas for free flow states one can detect stochastic distributions typical for independent elements, for congested states the strong psychological linkage among crowding agents leads to the *strong systemization of the ensemble*. Recently, these microscopic phenomena are measurable, which opens new possibilities for inspecting a local behavior in human groups.

MATHEMATICS



Mathematical methods applicable for simulating of Agent Systems with socio-physical interactions are as varied as the socio-dynamics itself. Besides macroscopic traffic models (applying the theory of non-linear partial differential equations) the main accent now is concentrated on microscopic transport schemes. Those numerical (or analytical) approaches describe an associated agent's dynamics by means of physically-inspired interaction rules and stochastic randomizations. Indeed, quality of such models should not be evaluated by macroscopic correspondences with an empirical reality only. As a second resort, the important quality-criterion is represented by a study of quantitative similarities between statistical properties of traffic micro-variables (individual velocities, accelerations, gaps among vehicles and so on). Recently, theory of headway- or clearance-distributions of particle ensembles is a subject of intense interest of scientists. Mathematical researches for socio-physical systems of agents interacting via finite-ranged interconnections have revealed surprising regularities in their inner micro-structure.

Furthermore, general *theory of the statistical rigidity* have introduced certain powerful methods on how to evaluate theoretical probability-models for rigorous estimations of socio-physical distributions.

All these innovative approaches have emerged from the *Random Matrix Theory*. Such a modern mathematical discipline deals with statistical properties of unfolded spectra in matrices whose elements are chosen from an arbitrary statistical distribution. Many recent papers suggest a link between classes of random matrices and systems with socio-physical ties. However, the search for deeper connections still continues…

MICRO SCALE

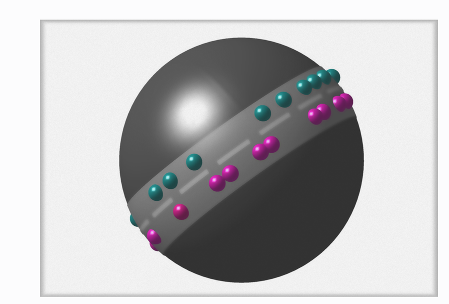
Stochastic properties of micro-quantities (i.e., quantities belonging to an individual agent) reflect a current macroscopic status of Agent Ensemble. As is well known, even a slight change of position in the so-called *fundamental diagram* (the dependency between the flow of agents and their density) may lead to substantial changes in distributions of individual quantities. It is therefore natural to examine the evolution of such distributions and to search for their connection to decision-making process working in the background.

Mathematical predictions for vehicular (or pedestrians) headways can be derived with the help of an appropriate socio-physical scheme, which represents an alternative formulation of one-dimensional thermodynamic gas whose particles are repulsed (attracted) by finite-ranged potentials.

The general theory of the above-mentioned gases is well researched in the pivotal article *"Equilibrium distributions in thermodynamic traffic gas"* published in 2007 in Journal of Physics A: Mathematical and Theoretical. This article has opened up new theoretical possibilities to predict not only headway distributions but also advanced statistical properties (e.g., the statistical rigidity) of Agent Ensembles.

In addition, some current papers have pointed to the fact that the micro-structure of some Agent Systems corresponds (under certain conditions) to the spectral properties of certain types of random matrices. Random Matrix Theory thus naturally became gray eminences standing in the background functioning of socio-physical interactions.

SIMULATIONS



Scientific approaches to socio-physical modeling are quite varied. Although macroscopic models are still widely used, in recent years, the preferred model-algorithms are based on the dynamics of individual agents. The indisputable advantage of these models is the ability to compare their micro-structures with empirical micro-variables measured on highways. Such comparisons enable a continuous improvement of the quality of models, which is, without any doubt, beneficial to the whole area of socio-physical modeling.

Macroscopic models are mathematical models that formulate the relationships among agent flow characteristics like density, flux, mean speed of an agent stream, etc. One of the most famous macroscopic traffic models is the Lighthill-Whitham model. Basic tenets of this model were published in 1955 in the article *"A theory of traffic flow on crowded roads."*

In contrast to macroscopic models, microscopic agent flow models simulate single agent units, so the dynamic variables of the models represent microscopic properties like the position, velocity of individual agents, or acceleration. The microscopic traffic models can be divided into several groups:

1) time-continuous models (also known as car-following models) defined by ordinary differential equations;
2) space-discrete models (also known as cellular automata) whose dynamics is given by update rules;
3) continuous thermodynamics models whose evolution is described by socio-physical forces and stochastic disturbances, and finally;
4) matrix models defined by purely mathematical instruments known from Random Matrix Theory.