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Understanding anomalous stochastic states in vehicular microstructure

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Abstract

Studying recent empirical traffic data, we show surprising statistical anomalies in the traffic microstructure that can not be explained by current scientific approaches used in physics of traffic. We quantify these anomalies mathematically and explain their cause. By means of particle gas, which represents a specific version of the traffic model described by the general Langevin equation, we show that all these anomalies can be explained by an occurrence of attractive force components in the model. This approach (in addition to the explanation of the statistical properties in vehicular microstructure) also makes it possible to detect conditions in real traffic flows, under which the attractive stimuli are considerable. The detected area, where the presence of strong inter-vehicle attraction stimuli is fully manifested, perfectly matches the reality of traffic.

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1. Vehicular flow as stochastic particle system

From mathematical viewpoint, an arbitrary state of vehicular traffic on one-lane expressways is equivalent to the well-known theoretical concept of balanced particle systems. It is represented (see Treiber et al. (2009); Krbalek (2007)) by the stochastic ensemble of ordered point-like particles located on a half line with a reference particle ($k = 0$) positioned at ξ_0. Instead of locations $\xi_0 < \xi_1 < \xi_2 < \ldots$ the particles are described by mutual headways $x_k := \xi_{k+1} - \xi_k$ and multiheadways $y_k := \xi_{k+1} - \xi_0$. Level of stochastic perturbations in this system is driven by a parameter called the stochastic resistivity $\beta \geq 0$. This parameter is (in a classical physical interpretation) represented by $1/k_B T$, where T is a temperature and k_B is Boltzmann factor. Provided $\beta = 0$, i.e. for $T \to +\infty$, the systems shows a maximal degree of stochastic noise and corresponds, in fact, to the *Poissonian ensemble* of absolutely random and uncorrelated events. In such system, the probability that k particles lies inside the interval $(\xi_0, \xi_0 + L)$ is equal to $\mathcal{P}[N_L = k] = (\lambda L)^k \, \mathrm{e}^{-\lambda L}/k!$, which is the well-known Poisson distribution. At the same time, the probability density function for inter-particle headway X (understood as a random variable) is described by the exponential function

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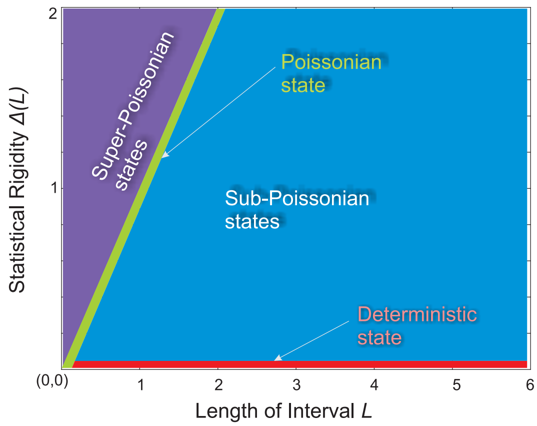

Fig. 1. Classification of states in stochastic systems according to the statistical rigidity.

$g(x) = \lambda\Theta(x)e^{-\lambda x}$, where $\Theta(x)$ is Heaviside unit-step function. For a scaled variant of such systems, when estimated value of headways $\mathtt{E}(X) = \int_{\mathbf{R}} xg(x)\,\mathrm{d}x$ is equal to one, one can choose $\lambda = 1$. Under this condition the variance $\mathtt{VAR}(X)$ is equal to one.

Consistently, in this presentation we consider the *scaled* variant of particle/vehicle systems. Empirical data analyzed in this work are therefore re-scaled so that the mean clearance (distance between a rear bumper of previous car and a front bumper of a reference car) is equal to one.

2. Classification of systems according to compressibility

Level of stochastic fluctuations of random variable N_L (number of particles occurring in the interval $(\xi_0, \xi_0 + L)$) used to be typically described (see Mehta (2004); Krbalek et al. (2009)) by the well-known instrument of Random Matrix Theory called *statistical rigidity (number variance)*. This continuous function is (for scaled particle systems) defined by $\Delta(L) = \sum_{k=0}^{\infty}(k-L)^2\mathcal{P}[N_L = k]$. It is not difficult to show that in Poissonian systems it holds $\Delta(L) = L$. Since there are no external reductions of stochastic fluctuations (like repulsion forces between particles, for example), Poissonian ensemble represents a system with the maximum possible degree of fluctuations. The function $\Delta(L) = L$ seems therefore to be a natural upper limit for statistical rigidity of all one-dimensional particle systems. Systems with reduced rigidity are therefore called *sub-Poissonian*, as value $\Delta(L) = 0$ indicates a *deterministic* systems without any fluctuations, where $\beta \to +\infty$ and $T \to 0_+$.

Although in general systems the rigidity $\Delta(L)$ has not a pure linear behavior, it shows, however, a noticeable linear asymptote $\chi L + \varkappa$ with the slope χ called *compressibility* and intercept $\varkappa$ called *deflection*. In systems where particles are repulsed by inter-particles forces the compressibility is (in contrast to Poissonian systems) suppressed, i.e. $\chi < 1$. Super-Poissonian systems, whose statistical fluctuations exceed fluctuations observed in systems with totally independent/uncorrelated events, are therefore very rare (and almost unmentioned in the literature). For such systems it holds $\chi > 1$.

3. Anomalies in vehicular microstructure

Statistical analysis of empirical traffic data (a course of statistical rigidity detected for homogeneous phase segments) confirms that states of traffic flow in a main lane remains in the sub-Poissonian territory. However, much more interesting behavior is detected when analyzing fast-lane data. Whereas some fast-lane samples lie in the territory of sub-Poissonian states, as well, the statistical rigidity of other samples surprisingly intersects the territory of super-Poissonian states. The latter represents data samples measured for lower traffic densities, when the traffic is in the free or transition phases. More detailed analysis of the real-road rigidity has been carried out (according to the 3s-

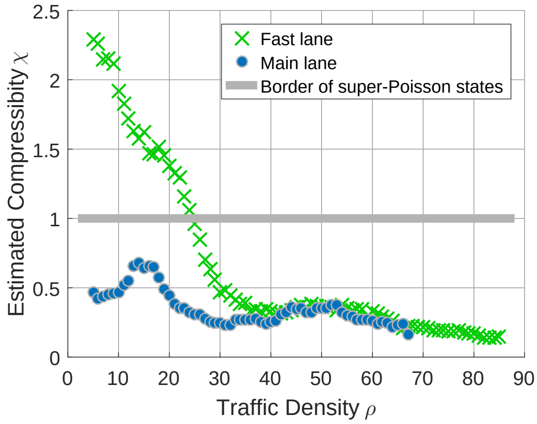

Fig. 2. Stochastic compressibility of freeway data.

segmentation procedure – see Krbalek (2018)) as follows. First of all, a fixed phase segment Ψ has been defined as a density band $\Psi = [\varrho, \varrho + 5\, veh/km]$, where the traffic density ϱ is a parameter. From the entire record it has been then extracted data sub-samples (with the sampling size equal to 50) belonging to the given phase segment Ψ. For the resulting $\Psi-$adjoint set of traffic micro-quantities (clearances, headways, velocities, and so on) it has been calculated a course of statistical rigidity. With help of robust regression methods, eliminating the onset trend (curvilinear behavior of rigidity near the origin), we have quantified the value of compressibility χ. Changing the traffic density ϱ in a segment Ψ we have obtained a dependency $\chi = \chi(\varrho)$ describing of how rigid the traffic streams is. Results of this analysis are plotted in figure 2. They confirm that $\chi < 1$ for all segments of a main lane. The same behavior is visible for fast-lane data extracted from a congested traffic phase (for densities larger than 25 veh/km). Contrariwise, free-flow compressibility exceeds the border value $\chi = 1$, which means that associate traffic states are super-Poissonian. These anomalous states are planned to be explained in this contribution.

4. Explaining the super-Poissonian states (Conclusion)

The performed test reveals very interesting discrepancies between main-lane and fast-lane vehicular streams. However, for densities greater than 40 veh/km it can hardly be distinguished between both lanes. The reason for such behavior is following. At high densities, lane changes (related to overtaking vehicles) are extremely improbable. Both lanes are significantly synchronized and the flow in them is equivalent more or less. A completely different situation occurs for flow at lower densities, where significant freedom for drivers' decisions (and a possibility to choose between a more sporty driving style in a fast lane or a more moderate style in a slow lane) causes inhomogeneous division of vehicles into two lanes. This situation leads to more rigid arrangement of vehicles in a main lane (reflecting stronger orderliness of vehicles). On the other side, in a fast lane one can recognize behavior whose stochastic level is more intense than for absolutely stochastic system of randomly distributed particles (Poissonian system). Since compressibility of all particle systems, where interaction stimuli are purely repulsive, shall not exceed a unit value (as mathematically proven in the theory of short-ranged particle systems), above-limit compressibility values can be explained by the presence of an attractive force component in the system. This attractive force component arises as a result of overtaking maneuvers.

References

Treiber, M., Helbing, D., 2009. Hamilton-like statistics in onedimensional driven dissipative many-particle systems. Eur. Phys. J. B 68, 607.
Krbálek, M., 2007. Equilibrium distributions in a thermodynamicaltraffic gas. J. Phys. A: Math. Theor. 40, 5813.
Treiber, M., Kesting, A., Helbing, D. 2006. Understanding widely scattered traffic flows, the capacity drop, and platoons as effects of variance-driven time gaps. Physical Review E 74, 016123/01.
Mehta, M.L., 2004. Random matrices (Third Edition). New York: Academic Press.

Krbálek, M., Šeba, P., 2009. Spectral rigidity of vehicular streams (random matrix theory approach). J. Phys. A: Math. Theor. 42, 345001.
Krbálek, M., Krbálková, M., 2018. *3s-Unification for Vehicular Headway Modeling,* Proceedings of Stochastic and Physical Monitoring Systems Conference 2018, 69. (arXiv: https://arxiv.org/abs/1811.05325)